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MECHANICS RESEARCH DEPARTMENT

INTERIOR BALLISTICS
OF
LIQUID PROPELLANT SMALL ARMS
ORDNANCE PROJECT TS1-11
CONTRACT DAI-11-022-ORD-(P)8

FINAL REPORT
AMF PROJECT MR1025

for

FRANKFORD ARSENAL
PHILADELPHIA 37, PENNSYLVANIA

ORIGINATED
AMERICAN MACHINE & FOUNDRY CO.

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INTERIOR BALLISTICS
OF
LIQUID PROPELLANT SMALL ARMS

This final report summarizes the research performed on Ordnance Contract No. DAI-11-022-ORD-P(8) entitled "Interior Ballistics of Liquid Propellant Small Arms". This work was performed under the technical supervision of Frankford Arsenal during the period August 1954 to 30 November 1955 and is identified as American Machine and Foundry Company's Project MR1025.

The Mechanics Research Department of the American Machine and Foundry Company is glad to cooperate with Frankford Arsenal in this research program, and any suggestions or comments on the work performed will be appreciated.

Personnel making substantial contributions to this project are:

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ABSTRACT

This is the final report on Contract No. DAI-11-022-ORD-P(8) entitled "Interior Ballistics of Liquid Propellant Small Arms". The report contains a summary of a literature survey of the liquid propellant field and the results of an interior ballistic analysis of experimental data.

Experimental data obtained by Frankford Arsenal on a caliber .60 weapon are used for this purpose and constants for use in the ballistic equations are determined. It is shown that a quadratic form function and a linear burning rate equation can be used to predict the maximum pressure and muzzle velocity with sufficient accuracy. The compromise values of the burning rate and form factor necessary for correlation are given. Comparison is made with solutions obtained with a cubic form function equation and a method for determining these form factors is reported. Data from rounds with various charge to projectile mass ratios are analyzed and show that the burning as indicated by a fictitious fractional web is unaffected by a charge to mass ratio variation of 2 to 1. A brief analysis is given indicating the apparent burning surface obtained by assuming the type of burning rate equation used for solid propellants but using a value of $n > 1$.

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INTERIOR BALLISTICS
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I. INTRODUCTION

Research on the use of liquid propellants and the development of suitable weapons using liquid propellants have been the subject of programs sponsored by all branches of the service for the past several years. These programs have ranged from fundamental studies on freezing points of various LP mixtures to the development of 40mm repetitive fire weapons. Both preloaded and injection systems have been investigated. The research activity at Frankford Arsenal has been directed toward the development of a preloaded monopropellant system using hydrazine-hydrazine nitrate in a caliber .60 weapon. Reproducible performance has been obtained with pyrotechnic ignition and the need for an adequate interior ballistic system has been expressed.

The Mechanics Research Department of the American Machine & Foundry Company was requested by the Office, Chief of Ordnance, through Frankford Arsenal, to conduct a research program leading to the development of an interior ballistic theory for liquid propellant small arms. This program was begun in August 1954 on Ordnance Project No. TSl-11 under Contract No. DAI-11-022-ORD-(P)8. Work continued on this contract until its termination November 30, 1955. Two simultaneous phases of the program were initiated. A literature survey was begun to keep abreast of other developments in the field and the interior ballistic analysis of data immediately available from Frankford Arsenal was made. The information obtained from these two investigations are included in this report.

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II. LITERATURE SURVEY

A survey of existing literature served to acquaint AMF personnel with the status of LP research and development to-date, and, therefore, the survey was continued for the duration of the contract. The survey was to serve as a possible source of ballistic data which would be analyzed and incorporated in the interior ballistic theory which would be developed in the second phase of the project. Unfortunately much of the research and development work was toward the development of operating guns of various calibers and on the chemistry of the various liquid propellants. Considerable amounts of the literature concerned caseless and regenerative feed systems which are of only cursory interest for this project. Thus the bibliography in Appendix A does not contain all of the many reports which were surveyed, but only those which contain the information pertinent to the interior ballistics of pre-loaded small arms.

A. Thermodynamic and Thermochemical Properties of Hydrazine-Hydrazine Nitrate

1. Products of Decomposition

It is generally considered that hydrazine completely decomposes into N_2 and H_2 . Any water present acts only as a diluent and does not participate in the reaction. The reaction when hydrazine nitrate is present may be shown as



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These reactions assume complete dissociation of any ammonia which may be formed as an intermediate reaction. Investigations at Jet Propulsion Laboratory (17) have assumed that ammonia is formed in the reaction $3\text{N}_2\text{H}_4 \rightarrow 4\text{NH}_3 + \text{N}_2 \rightleftharpoons 3\text{N}_2 + 6\text{H}_2$, and have shown that the maximum impetus is obtained when approximately 35 per cent ammonia formation occurs. Per cent or fraction of NH_3 formation refers to the fraction of the theoretically maximum amount of NH_3 which may be formed by the above reaction. Thermodynamic data are presented for several amounts of ammonia formation. Nozzle erosion data obtained by Armour Research Foundation (3) with a vented bomb have been used to indicate gas temperature in the throat of approximately 2000°K . This temperature is obtainable with the specific composition only if about 35 per cent ammonia formation occurs. These data give direct evidence of the probable composition of the gas during the actual ballistic cycle.

2. Impetus and Isochoric Flame Temperature

Values for the impetus and isochoric flame temperature for hydrazine nitrate water have been reported by Armour Research Foundation (5, 6) and the Jet Propulsion Laboratory (9, 17). Armour Research Foundation assumed no dissociation and no ammonia formation. JPL gave data with up to $2/3$ of the theoretical NH_3 formation. Impetus and flame temperature increase with nitrate addition from about 31×10^4 ft-lb per lb for N_2H_4 to approximately 46×10^4 ft-lb per lb at 85 per cent $\text{N}_2\text{H}_5\text{NO}_3$. Theoretical flame temperatures vary from 1180°K

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for N_2H_4 to 3375°K for 92.5% $N_2H_5NO_3$. Both impetus and flame temperatures are reduced by the addition of water. It has been shown (5, 17) that in the range of compositions generally used, 1/3 ammonia formation increases the impetus 3 to 5 per cent above the value calculated for no NH_3 formation. The isochoric flame temperature is more sensitive, increasing approximately 200°K. Hydrazine alone exhibits a maximum impetus with about 35 per cent NH_3 formation (17).

3. Ratio of Specific Heats and Co-volume

The ratio of specific heats is also dependent upon the products of combustion. Values for γ have been determined by Armour Research Foundation (6) for several monopropellant compositions ranging from 100 per cent hydrazine nitrate to 100 per cent hydrazine and for several different fractions of water. In these calculations it was assumed that no ammonia was formed. The values of γ vary from 1.241 for 0.8 mole fraction $N_2H_4 \cdot HNO_3$ with no water present to 1.381 for 100 per cent N_2H_4 . The presence of ammonia would tend to lower the values.

Several values of γ have been reported in literature. Jet Propulsion Laboratory (18) used $\gamma = 1.115$ in calculations for a 3.2 in. gun using a monopropellant of 71.7 per cent N_2H_4 and 28.3 per cent $N_3H_2NO_3$. This value seems extremely low and at best, would require that the products of combustion be almost entirely NH_3 and H_2O in order to obtain this low value. Kerr-

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Smith (46) uses a value of $\bar{\gamma} = 1.20$ for a liquid mixture consisting of 71 per cent hydrazine nitrate, 13.6 per cent H_2O , 6.7 per cent N_2H_4 , and 8.5 per cent NO_2 and products of only H_2O , N_2 and H_2 . Armour Research Foundation calculations would yield 1.34 and 1.245 for approximately the same mixtures. Adams (1) reported 1.20 for mixtures containing 54 per cent hydrazine and 37 - 41 per cent hydrazine nitrate, assuming 1 mole NH_3 formation for each mole N_2H_4 not oxidized by the nitrate.

The co-volume of the propellant gases is also a strong function of their composition. Jet Propulsion Laboratory (16, 18) and Armour Research Foundation (5, 6) use a theoretical value of 38 in^3 per lb. for low pressure equilibrium with large quantities of NH_3 formation. Kerr-Smith (46) uses 21.5 for a gas mixture assuming no ammonia present. Armour Research Foundation (3) also reports experimentally determined values of 30 for a 35 per cent N_2H_4 HNO_3 mixture and 35 for a 95 per cent N_2H_4 , 5 per cent H_2O mixture. These values were determined in a closed bomb at pressures between 10,000 and 30,000 psi. At this pressure level the effect of co-volume is small and appreciable error in the determination may result. Therefore, these values should only serve as a guide.

4. Linear Burning Rate

Determination of the linear burning or consumption rate of hydrazine nitrate has been carried out by several investigators

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including Adams and Stocks (2) in England and the United States Naval Ordnance Laboratory (40, 41) and Redel Inc. (24, 44) in this country. The method used is essentially standard and consists of measuring the time it takes the liquid in a glass tube to burn a given distance as denoted by the melting or breaking of fine wires stretched across the tube. Tubes of 3mm to 7mm have been used but the most extensive work has been done in 5.0 - 5.5mm tubes. An increase in consumption rate with tube diameter has been attributed to variation of reaction area due to the change in meniscus with tube diameter. The investigators have found that there is a critical pressure region in which the propellant will not ignite or sustain burning. The level and spread of region depends on the composition of the propellant. The upper value is 8500 psi for a 50 - 50 composition hydrazine-hydrazine nitrate solution and increases to 12,000 - 12,500 for a 25 per cent hydrazine nitrate mixture. In general the consumption rate increases with nitrate content and pressure. For each composition then has been found at least one region of negative slope when the rate sharply decreases with pressure. Once this region is past, the rate again increases but not necessarily at the same slope as before. Typical curves reported by Redel Inc. are shown in Fig. 1. The slope of the curve in the log log presentation denotes the exponent of the pressure for the consumption rate equations. This varies from approximately 2 to nearly 3 except for the higher nitrate mixtures at pressures of 15,000 psi or

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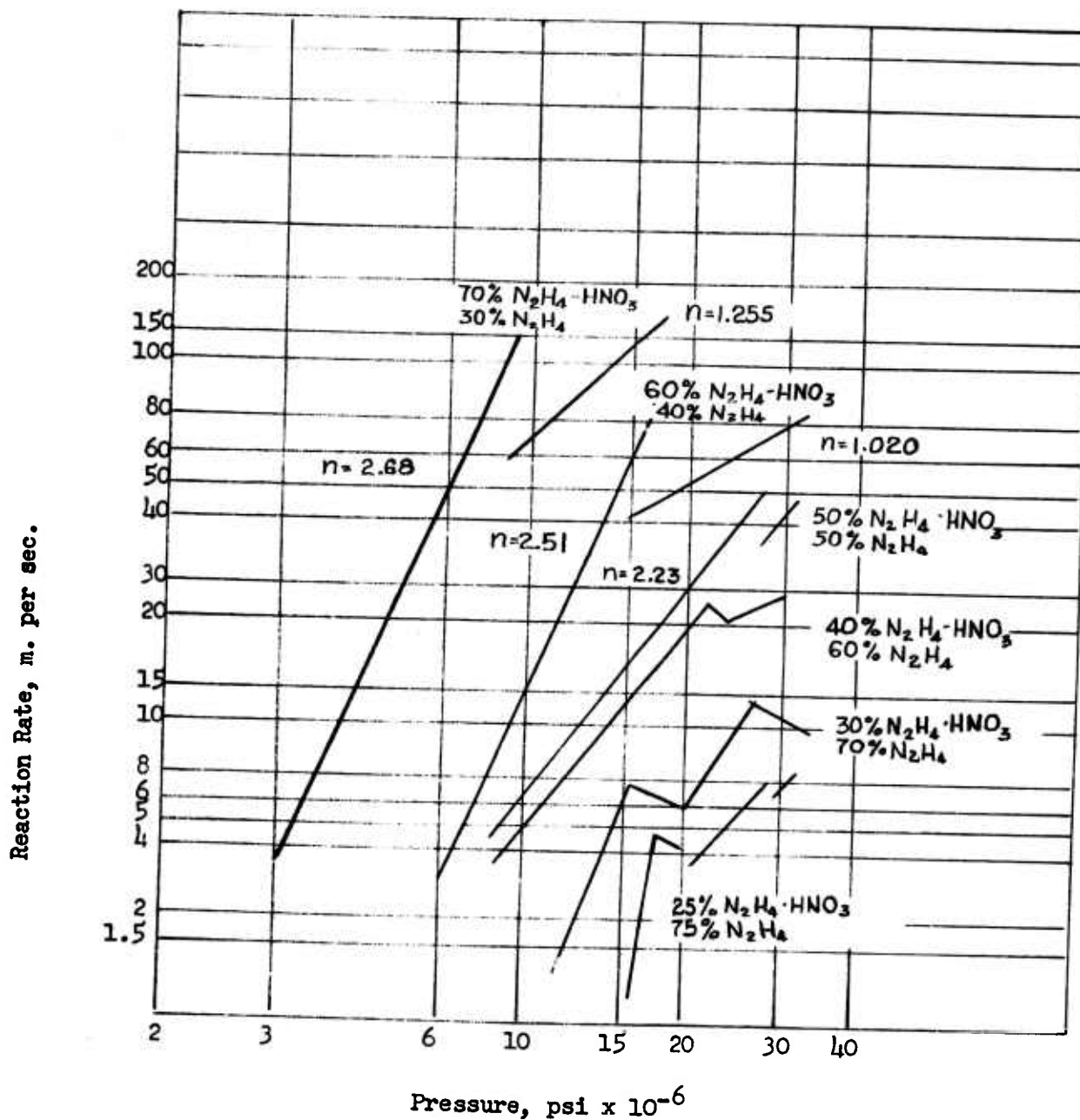


FIG. 1 BURNING RATE AS A FUNCTION
OF PRESSURE AND COMPOSITION (REDEL, INC.)
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higher. A slope of 1 to 1.25 is shown for these.

Navord Lab (41) reports consumption rates of about 5 in per sec for pressures around 4000 psi using a 5mm tube and 25 per cent nitrate. These rates are extremely high compared to Redel's work showing 5 in/sec at 25,000 to 30,000 psi for approximately the same composition.

Recently limited tests (30) have shown that increased surface area caused by using several capillary tubes together can double the consumption rate.

a. Effect of Storage

Redel Inc. (29, 30) reports decreased consumption rates for their standard composition upon storage. A decrease of 40 per cent in 3 months was reported for a sample containing aluminum strip. The control sample decreased 25 per cent but the sample containing stainless steel strip showed no decrease until after six months when a 30 per cent decrease was recorded; at this time the control sample would not ignite at 30,000 psi.

b. Effect of Viscosity

It has been shown (30, 40) that the viscosity effects the consumption rate. Redel Inc. was able to eliminate one region of negative slope from an established burning rate curve by increasing the viscosity with the addition of locust bean gum to the mixture.

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c. Effect of Initial Temperature

No consumption rate determinations have been reported at reduced temperatures. Experimental firing data at reduced temperature such as reported by Olin Mathieson (47) show that a pressure increase occurs below about 30°F and reaches a maximum at the freezing point of the mixture. Below the freezing point the pressures decrease, but become erratic. Pressures increase linearly with temperatures between 30°F and 140°F. How much of this effect is due to a change in burning rate is unknown.

d. Effect of Composition

Increased nitrate content increases consumption rate. Water addition tends to decrease the rate.

5. Optimum Composition

Olin-Mathieson Chemical Corp. conducted a firing program with a 20mm weapon to determine an optimum mixture for their conditions. It is reported (12) that 7.5 per cent water and 20 - 25 per cent $N_2H_4 \cdot HNO_3$ appear to give the maximum velocity to pressure ratio for that weapon.

B. Performance Characteristics

1. General

The general shape of the pressure-time record obtained from a high loading density liquid propellant gun differs markedly from the curve produced by the burning of the typical solid propellant

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charge. The rate of pressure rise is much more rapid than that of an equivalent solid propellant. The region of maximum pressure then takes on one of two general characteristic shapes. Ideally it flattens out into a plateau shaped maximum before decaying monotonically to zero. The second characteristic curve is a "saddle" shape consisting of two peaks. The first of these is referred to as the "ignition peak" and comes at the end of the sharp rise. A sharp pressure decrease forms a "valley" before the second peak after which the pressure decreases monotonically as in the ideal case. The cause of the peaks is not fully understood but are controllable to some extent. In general, an average of the two peaks results in a pressure close to what might be obtained on a "plateau" type round. They can both be of about the same magnitude or if the initial peak is high, the secondary peak is low and vice versa. This phenomenon is quite sensitive to ignition and somewhat affected by composition and/or consumption rate.

Typical of pressure records obtained from LP charges even in closed bomb studies is the presence of "hash" or high frequency vibrations on the record which may be of the same order of magnitude as the pressure signal itself. Low loading density guns give pressure traces similar to closed bomb traces. This might be anticipated since it is essentially a constant volume burning with negligible projectile travel prior to all burnt.

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2. Effect of Loading Density

Olin-Mathieson Chemical Corp. (12) in order to obtain evidence on the behavior of a low loading density charge, performed a large number of firings using an 8cc tests chamber with different sized barrels attached, and with different weight projectiles, but with the charge limited to a maximum of 2.0cc.

For hydrazine monopropellant with 5 per cent water and 5 per cent nitric acid, it was observed that for each charge, regardless of bore diameter or projectile weight, that the maximum pressure remained constant. This effect was reported consistent with the ballistic process attributed to the low loading density liquid monopropellant charge; namely that maximum pressure occurs before appreciable projectile movement takes place. Velocity was reported to vary in accordance with charge and projectile mass. Poudrier (44) at the U. S. Naval Proving Ground has reported experimental data indicating a minimum pressure occurring at about 10 per cent free volume. For a given charge, pressure is highest at high values of free volume (low loading density).

3. Effect of Additives

Tabular data were reported on the effects of additives by Olin-Mathieson Chemical Corp. (12) and Syracuse University (48). Olin-Mathieson Corp. attempted to reduce the hash on the pressure tracer by the use of additives. Of the compounds studied, zinc nitrate and ferrous chloride additions up to 1 per cent resulted

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in hash reduction without velocity degradation. Chamber pressure was increased appreciably, however, according to the report.

Syracuse University studied the effect of additives on freezing point depression.

4. Effect of Viscosity

In order to test the effect of propellant viscosity on gun performance, Detroit Controls Corporation (7), added small quantities of locust bean gum (up to 1.5 per cent by weight) to the propellant, increasing its viscosity from 2.9 to a maximum of 7200 poises. While there was no improvement in gun performance, the change in viscosity of the propellant had a marked effect on the combustion process. The most apparent change is that ignition has been adversely effected, and erratic combustion has followed. In many of the firings, particularly those firing the most viscous propellants, much of the propellant was not burned and remained in the barrel and combustion chamber after firing.

See also previous section on linear burning rate.

5. Effect of Chamber Geometry

Both Frankford Arsenal (42) and Reaction Motors (49) have reported work on chamber geometry effects. The Frankford Arsenal work is discussed in Section B643b on ignition. Reaction Motors report results from different chamber configurations and special grids for promoting turbulence after ignition. It is shown that chamber geometry has a definite effect on pressure and velocity.

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6. Effect of Ignition System

a. Pyrotechnic System

The experimental studies on ignition of liquid monopropellant by pyrotechnic means were directed at optimizing the ignition system from the following considerations:

1. Venting, location and size of vents
2. Penetration
3. Ignition Pressure

When a pyrotechnic is used as a primer, it is usually located in a tube with either radial and/or axial spit holes. The propellant surrounds the primer tube and is ignited from the flames issuing from the spit hole due to the burning of the pyrotechnic. The ignition of the priming material is usually accomplished by using a primer cap containing a very small amount of pyrotechnic which is sensitized by either a mechanical blow or by an electrical method.

(1) Venting Studies

(a) Effect of Varying the Position and Number of Radial Ignition Regions

Frankford Arsenal (33) reported that the use of optimally spaced multiple radial injection regions yield more plateau shaped p-t curves than were obtained with single regions. In addition, it makes possible the achievement of velocities comparable to those obtained with axial systems.

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(b) Effect of Varying Radial Ignition Orifice Area

It was observed by Frankford Arsenal (33) that the peak pressure and velocities decreased as the area was increased with the optimum p-t shape occurring in the group having the greatest orifice area. It was also, noted, that the peak pressures in all cases occurred in the ignition position of the curve.

(c) Effect of Radial Vent Locations

Olin-Mathieson Chemical Corp. (12) studied the effect of radial igniter flame on ignition and the overall ballistic characteristics. A standard 8cc test chamber was used with a 150 grain, caliber -.30 projectile. A constant length primer flash tube was used extending the entire length of the chamber. With this configuration, the location of the igniter flame could be varied by varying the position of the radial flash holes along the tube, without affecting the volume or configuration of the gun chamber or the tube itself.

In all tests conducted, the maximum chamber pressure increased as the position of the igniter flame was moved from the rear to the forward section of the chamber.

The pressure-time records change from an approximate plateau shaped curve with rear end ignition to a sharply peaked curve with front end ignition.

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(d) Effect of Position of Axial Ignition Orifice in the Fuel Column

Frankford Arsenal (33) reported that initially it had been observed that the ignition peak pressure decreased as the region of axial ignition was shifted forward in the fuel column. The results reported show that contrary to expectations, the intermediate length yielded the highest ignition pressure and the optimum pressure-time shape.

In cylindrical combustion chambers, as used in this work, axial ignition systems, in which the primer gases are vented through a hole drilled in the end of the flash tube, are more efficient than radial systems in which primer gases are vented through holes drilled in the flash tube wall. The qualification on chamber geometry is inserted since it is considered possible that the difference is due to loss of ignition energy to the chamber walls in some of the radial tests.

(2) Penetration

(a) Effect of Primer Vent Area to Cavity Volume Ratio, $\frac{S_v}{V_a}$

In the case of RedelInc. Z-1 pyrotechnic reported by RedelInc., (31) penetration increases rapidly with increasing ratios up to 0.1. Beyond this value, it does not change appreciably except for a pronounced dip at

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0.224. This point was checked by a repeat firing and found to be valid. With FFG, black powder, on the other hand, penetration increases less rapidly than for Z-1 and is not appreciably effected by increasing values of the ratio above 0.4.

(b) Effect of Density of Loading, Δ_1 ,

Redel Inc., (31) reported that the displacement of the primer products is not dependent upon, Δ_1 , when S_v/V is held constant, although penetration was least for all loading densities at lowest values of S_v/V .

It can be summarized from (31) for the weights of primer materials Redel Inc. investigated, that the maximum jet penetration during 0.3 to 0.5 msec is from 1.4 to 1.75 inches for Redel Z-1 and 0.9 to 1.35 inches for FFG. The optimum S_v/V value is about 0.5 in^{-1} and the optimum Δ_1 is about 0.5 gm/cm^3 .

(3) Ignition Pressure

Jet Propulsion Laboratory (11) stated that one of the main problems in the ignition of a monoliquid propellant has been to limit the ignitor pressure to a reasonable high value. In general, the ignition pressure has tended to be too high.

(a) Effect of Additives

Jet Propulsion Laboratory (11) found that for a constant charge, varying the amount of RFNA or WFNA igniter

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is not a satisfactory means of limiting the ignition pressure. However, by adding water to the hydrazine the ignition pressure could be limited in a narrow range of water concentration.

(b) Effect of Chamber Geometry

Frankford Arsenal (42) observed while conducting tests with cased and caseless rounds, that the ignition peaks were consistently much higher for the caseless than comparable cased firings. It was postulated that two basic differences might contribute to increased ignition pressure encountered. The first of these lies in the slightly larger diameter of the caseless chambers which might permit more propellant to be ignited by the initial primer exhaust per unit length of penetration. A second lies in the difference in the radii of curvature of the rear of the case and of the caseless breech plug obturate. The curvatures are such that the amount of propellant to the rear of the axial orifice in the caseless breech plug is 2.3cc, whereas the corresponding volume in curved configuration is 6.1cc.

(4) Pressure Variations

The U. S. Naval Proving Ground at Dahlgren (15) postulated, from experimental observations, that the quantity of energy

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supplied to the propellant by the igniter, the rate of delivery of this energy, the distribution of this energy to the propellant, and the amount of turbulence created in the propellant by the igniter must be controlled within narrow limits if satisfactory uniformity is to be obtained.

(5) Perchlorate Ignition

Olin-Mathieson Chemical Corp. (14) found that they could obtain the best ignition of 38 grams of liquid propellant, by using 1/2 gram ammonium perchlorate. When perchlorate is used as a booster in the gun, it is partially decomposed by this primer flame and the resulting mixture of oxygen rich flame plus decomposed perchlorate is blasted into the hydrazine in the vicinity of the primer.

(6) Theoretical Considerations

Redel Inc., (32) derived at the following relation to describe the dynamics of a pyrotechnic jet:

$$V = \frac{2(p_1 - p_2)}{K_s} \frac{\exp \left[2 \sqrt{\frac{(p_1 - p_2) K_s}{2 \rho L^2 c^2}} t \right] - 1}{\exp \left[2 \sqrt{\frac{(p_1 - p_2) K_s}{2 \rho L^2 c^2}} t \right] + 1}$$

where:

- K_s = coefficient of total resistance (wall and orifice) for steady flow at the instantaneous velocity
 V = velocity of jet
 C = unsteady flow constant

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ρ = fluid density
L = conduit test length
p = pressure
t = time

b. Spark Ignition System

Considerable effort has been expended in the field, not only in developing a suitable spark ignition system, but in theoretical analyses of the mechanism of sparking and ignition from spark sources.

(1) Theoretical Considerations

Stanford Research Institute (4) is conducting a liquid propellant program in which they are investigating the mechanism by which a spark ignites the monopropellant.

The framework of their hypothesis is provided by two experimental observations; that the proximity of the walls to the spark makes ignition easier, and that the shock waves are provided by a very high energy spark. The rate of chemical reaction caused by the spark will have an influence on the behavior of the outward moving shock wave by virtue of its energy contribution to the expanding piston. The reaction zone may travel with the head of the shock wave, since the shock strength is so great that the temperature in the shock front is appreciable. However, unless a steady state detonation wave is set up, the spherical shock wave

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will attenuate as it moves outward and the reaction will proceed at a slower rate behind the shock front in the region of elevated pressure. The behavior of the shock wave will be determined by the rate of input of energy to the piston by the spark and by the chemical reaction. It is supposed that if the pressure in the neighborhood of the reaction falls below the minimum pressure for burning, the reaction will cease. It is further supposed that if rigid walls are situated at a suitable distance from the spark, the increased pressure in the first reflected shock will sustain the pressure in the neighborhood of the reaction zone for a time sufficiently long that the critical volume of liquid can have an opportunity to react.

The hydrodynamic problem then is one of the calculation of the behavior of a spherical shock wave generated by a spherical piston moving outward at a rate determined by the rate of delivery of spark energy and the rate of reaction of the propellant.

Redel Inc., (27) hypothesized that the mechanism of electrical action consists of the following three steps:

1. heating and vaporization of a portion of the liquid,
2. ignition of the vapor in the arc,
3. additional energy used to promote turbulence and ionization.

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As soon as the switch is closed, a potential is established across the spark gap. As the solution is heated, the resistance of the liquid decreases, and the current flow increases.

At some time the path between the electrodes contains a quantity of vapor such that the electrical flow changes in character from a flow of electrons through a liquid to a flow of ions through a gas phase, constituting an arc. This can occur only if the voltage at this instant is still above minimum spark voltage for that gap and gas constant. As this arcing process is essentially a high temperature one, a portion of the propellant vapor is raised from the boiling temperature to the decomposition temperature. An unknown quantity of the available energy is employed in this step and is believed to be very small.

After the arc is formed, the energy remaining in the system is poured through the arc. This continues until the voltages at the electrodes drops below the value required to maintain an arc across the gap. The arc then dies leaving a small residual of energy in the system. The characteristics of the electrical system determine the amount of residual energy. The energy being put into the arc is used in the third step of the mechanism in heating of the reaction product gases, dissociating product gases at high temperature, bringing additional propellant vapor

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to decomposition temperature and even vaporizing additional propellant. These actions are all exhibited as the creation of turbulence around the spark gap.

Redel further states (20) that experimental data point to the conclusion that the greater part of the spark discharge energy is in the formative phase and/or line loss. It is felt, however, that the majority of energy in the spark phase is available to promote turbulence in the system. Also a comparison of pyrotechnic and spark ignition, as applied to monopropellant initiation indicates that the type of ignition is not important so long as a certain minimum energy is reached.

University of Michigan (37) found that when an impulse potential is applied to a set of electrodes submerged in a conducting fluid, the current discharge takes place in two distinct phases. The first phase, called the "formative phase", is characterized by a high voltage drop across the electrodes and a low current flow. The second phase called the "arc phase", is characterized by a low voltage drop across the electrodes and a high current.

The energy delivered to the monoliquid propellant during the arc phase of discharge does not seem to be the sole criterion for ignition. Four ignitions were obtained with arc energies between 4.5 and 8 joules, and yet no ignition was obtained on the other tests when the arc energy was 8 to

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10 joules energy. Thus it appears that the formative energy and total spark energy dissipated, as well as the arc energy, all bear some relation to the ignition process.

(2) Factors Affecting Spark Ignition

It was found by Olin-Mathieson Chemical Corp. (10, 12), that ignition of hydrazine would not occur when a spark was discharged beneath the liquid surface, even with the chamber completely filled with liquid propellant and sealed by a heavy aluminum diaphragm. Ignition occurred only when the chamber was prepressurized with nitrogen to approximately 2000 psi. (See section on electrodes).

(a) Plenums

Hydrazine and hydrazine-hydrazine nitrate mono-propellant mixtures have been ignited successfully by means of a single spark discharge without prepressurization by simply confining a small quantity of propellant in a plenum chamber surrounding the electrode. A spark plug was used incorporating a plenum volume, and the 0.30 caliber test gun was fired with the system using the propellant mixture containing 60 per cent hydrazine, 33 per cent hydrazine nitrate and 7 per cent water. The minimum spark energy required for ignition was 6 joules (3 mfd condenser to 2000 volts).

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(b) Formative Energy

According to the University of Michigan (36), it seems fairly definite that successful ignition can be obtained with an all formative discharge. Twelve gun firing tests were made with all formative and 10 of these resulted in successful ignition. The minimum energy for ignition seems to be slightly below 15 joules for the particular system being used, which is the same minimum energy as for the same standard formative-arc discharge. The peak power delivered during the all formative is in the same range as that obtained in the formative-arc discharge since the latter has its peak during the formative period.

(c) Electrodes

Univeristy of Michigan (38) showed, in a 10 ml bomb, completely filled with hydrazine-hydrazine-nitrate liquid propellant, that in order to obtain ignition it is necessary to cover all but the tips of the electrodes with an insulating material to prevent an undue amount of stored energy from being dissipated by conduction during the formative stage.

A particular difference between blunt and pointed electrodes was the amount of scatter in the energy data (35). For any given spark gap, the energy obtained with blunt electrodes was considerably more erratic, particularly in the formative phase.

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(d) Physical Properties

In comparing the changes in formative energy with other properties of the propellant, it was found that electrical conductivity, viscosity, and formative energy are all affected proportionally by changes in temperature (39). By utilizing various strengths of potassium chloride solutions and maintaining a constant propellant viscosity, it has been shown that an exponential relation exists between formative energy and conductivity. This same sort of relationship holds true for hydrazine propellant, but the exponent is not the same.

It has been observed, in spark discharge tests made with blunt and pointed electrodes that both energy and duration appear to increase slightly with increasing viscosity (39).

(e) Voltage

Initially it was determined (8) that the release of approximately 100 joules of electrical energy from a condenser bank was sufficient for ignition. Tests were made at both 2000 and 4000 volts to determine the effects of capacitor voltage on round performance while maintaining constant energy in the capacitors. No appreciable change in performance was observed other than a slight increase in peak combustion chamber pressure, accompanied by a minor improvement in reproducibility, in tests made at 4000 volts.

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C. Interior Ballistics

It is necessary, in any interior ballistic analysis, to consider the following five relations:

1. Energy Balance
2. Equation of State
3. Equation of Motion
4. Propellant Burning Law
5. Form Function

The first three relations are essentially the same for any type analysis. However, items 4 and 5, the burning law and form function, can differ considerably.

JPL (16) presented a method of interior ballistics for use on a high speed calculator (REAC) in order to correlate information on

1. initial burning characteristics (ignition), including effect of shot start pressure and effective initial burning rate,
2. form function, the mode of propellant break up and resulting change of burning rate,
3. effective impetus,
4. effective loss to barrel
5. pressure dependence to burning rate.

Heat loss was considered to be proportional to the kinetic energy of the projectile, as accounted for in solid propellant interior ballistics.

The burning law used, was of the type commonly applied to the burning of solid propellants, the fraction of web unburned is proportional to pressure raised to a power, as

$$D \frac{dr}{dt} = -B P^x$$

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where:

α = pressure exponent

P = breech pressure (assumes combustion takes place at breech pressure)

β = burning rate coefficient

D = web size

f = fraction of web unburned

Since liquid propellant has no web, in the solid propellant sense, the quantities f, β and D were redefined in terms of the physical properties of the liquid. Thus, D was defined as the quotient of the liquid volume divided by the surface area exposed to the igniter. Then if β is the usual burning rate coefficient, $D(df/dt)$ will be the linear burning rate normal to the surface, at least initially. Since D can be obtained empirically through the ballistics, β can therefore be determined.

It was postulated, that in order to obtain a flat pressure curve in a liquid gun it is necessary to have initially regressive burning followed by progressive. The form function used to obtain this condition was taken as

$$\phi = (1-f) (1+\theta f) (1+\theta_1 f)$$

where:

ϕ = fraction of propellant burned

θ and θ_1 are constants and $\theta\theta_1 > 0$ for the conditions mentioned.

After the propellant has completely burned, the expansion was considered as adiabatic.

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Computations were made using a quadratic form function ($\theta_1 = 0$) and assuming that burning occurs at space mean pressure. These computations showed that dependence of the various ballistic parameters with variations in each other and other dimensionless parameters. However, the results were too preliminary to draw any significant conclusions.

Redel (28) presents a method whereby the experimental pressure time curve is analyzed to determine the relation between propellant burned, propellant burning surface area, temperature, velocity and travel and time. Although this method may present insight into the values of the various ballistic parameters, at the conditions at which the experimental data was taken, in its present form it cannot be used as a method for weapon design.

The method presented, does utilize the necessary relations of energy balance, equation of motion and equation of state. However, unconventional interior ballistic notation is used.

The total heat loss was evaluated from the muzzle conditions, and then were inserted linearly with time. The internal energy was assumed to be either a linear or a quadratic function of temperature. An equation of state is used in which gas imperfections are accounted for by a compressibility factor, Z , rather than the usual covolume correction.

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JPL (18) also reports a method for calculating a system assuming a linear pressure rise, a constant pressure period at the maximum until all burnt gas an adiabatic expansion. Equations are presented for each of four periods in the ballistic cycle. This has been achieved by forcing the desired pressure function into the ballistic equations, and actual pressure functions only approach the desired conditions at best.

Redel Inc., (29) reports a more radical treatment of the burning by studying the propellant break-up. Use is made of Taylor's stability theory to determine the minimum and most probable droplet sizes. An attempt is made to take into account the surface tension and viscosity effects. Reacting surface areas of 60 times the chamber cross-section are calculated.

III. DEVELOPMENT OF INTERIOR BALLISTIC THEORY

A. Basic Equations

An interior ballistic calculation for either a solid or a liquid propellant weapon involves the simultaneous solution of four basic equations.

The energy balance equation

$$\frac{NRT_0}{\gamma - 1} - \frac{NRT}{\gamma - 1} = (1 + \beta) \frac{m'v^2}{2} \quad (1)$$

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The Abel equation of state

$$P \left[A_x - \frac{(C-N)}{\rho} - N\gamma \right] = NRT \quad (2)$$

The equation of motion

$$AP = m' \frac{dv}{dt} \quad (3)$$

The burning rate equation

$$\frac{dN}{dt} = \rho S(B/2)P^n \quad (4)$$

The first three equations, well known in solid propellant ballistics, can be applied directly to liquid propellants with little or no modification. The burning rate equation is based on the interfacial burning surface, S , which will vary during burning. With solid propellants, there is a constrained geometric surface and a definite relation exists between the surface exposed to burning and the fraction of the propellant mass consumed. This relationship is generally referred to as the form function. With liquid propellants, there is no surface constraint and the interfacial surface may vary considerably without regard to the fraction of propellant mass consumed. Thus the evaluation of a suitable expression for the burning equation constitutes the major problem in the liquid propellant ballistic theory.

For solid propellants the form function is expressed as a function of the fractional web, f , which is the ratio of the characteristic web length of the grain to its initial value. For liquid propellants the fractional web can have no such physical significance. However, it is convenient to

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consider it as a continuous variable which is initially "1" and monotonically decreases to zero at "all burnt". Using this notation

$$S = \frac{-2C}{W} \frac{1}{1+x} \quad (5)$$

and the Eq. (4) becomes

$$-W \frac{df}{dt} = BP^n \quad (6)$$

If it is determined that the form function varies with ballistic parameters evaluation of S must be performed from Eq. (4).

The progressivity of the burning is determined by the change of the interfacial surface and can be easily determined by Eq. (5).

The literature survey has shown that the following factors may affect the burning rate;

1. chamber geometry
2. loading density
3. ignition system
4. propellant properties
5. expansion rate

Reaction Motors Inc., (49) has shown that pre-ignition agitation of the propellant does not affect the ballistics.

B. Working Equations

In developing ballistic equations for solid propellants, it has been the practice to assume that the pressure exponent, n, in the burning equation is unity. This simplification is justified for solid pro-

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pellants since n varies between 0.7 and 1.0 for most propellants. Data on liquid propellants have indicated that n may be between 2 and 3 for the hydrazine-hydrazine nitrate monopropellant compositions generally used. However, it was believed, that in the interests of simplicity an interior ballistic theory based on a linear burning rate equation should be attempted. Therefore, taking $n = 1$, the four basic equations are combined into the general ballistic equation in terms of the following dimensionless parameters:

$$I = \sqrt{\frac{m^0}{CF}} V, \text{ dimensionless velocity} \quad (7a)$$

$$\lambda = x/x_0, \text{ dimensionless travel} \quad (7b)$$

$$\psi = \frac{2P}{\Delta_0 F}, \text{ dimensionless pressure} \quad (7c)$$

$$\sigma = \sqrt{\frac{m^0 CF}{A}} (B/W), = \sqrt{CF/m^0} 1/V_b, \text{ dimensionless burning rate constant} \quad (7d)$$

$$\tau = (1/2x_0) \sqrt{\frac{CF}{m^0}} t, \text{ dimensionless time} \quad (7e)$$

The resulting ballistic equation becomes

$$\left(\frac{\tau}{2} - 1\right) I^2 = \theta - 1/2 \left[\lambda - \frac{\Delta_0}{P} - a \Delta_0 \theta \right] \frac{d(I^2)}{d\lambda} \quad (8)$$

where:

$$\theta = \phi(f) \quad (8a)$$

$$f = f_0 - \sigma I \quad (8b)$$

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$$\psi = \frac{d(I^2)}{d\lambda} \quad (8c)$$

$$d\tau = dI/\psi = d\lambda/2I \quad (8d)$$

$$a = \eta - 1/\rho \quad (8e)$$

$$(\bar{\gamma}-1) = (1+\beta)(\gamma-1) \quad (8f)$$

β = heat loss proportionality factor.

The system of Eq. (8) may be solved if the form function Eq. (8a) is given. Solution of this system for a quadratic form function in f is well known. It will be shown later that a cubic form function best describes the burning for the plateau shaped pressure time curves, although experimental data can be closely approximated with a quadratic function by modifying certain other parameters. Solutions for both cases are reported.

C. Idealized Pressure Time Curve

Experimental data have shown two fundamental types of pressure-time relationship: the saddle shape and the plateau shape. It has been assumed that the plateau type curve which consists of a nearly linear pressure rise to maximum or close to maximum followed by a period of nearly constant pressure before a monotonic decrease during the latter part of the cycle is the desirable uniform burning pattern, and the saddle shape in which two peaks are obtained constitutes variation in the burning due to the effect of various parameters. The

plateau type of curve is idealized and expressions for the surface function are derived in Appendix B, assuming a linear pressure rise and then a constant pressure until all burnt. This analysis shows that the variation of surface with fractional web, f , can be expressed by two straight lines, as shown in Fig. 2, representing constant progressivity in each region. The plateau region has the lower progressivity as would be expected. The two solid straight lines may be closely approximated by a parabola as indicated by the dotted line. Based on conventional formulation of form functions, any straight line in Fig. 2 represents a quadratic form function while a parabola with the directrix oriented parallel to the f - axis represents a cubic form function of the form

$$\phi = k_0 - k_1 f + k_2 f^2 + k_3 f^3 \quad (9)$$

where: k_0 , k_1 , k_2 and k_3 are constants.

An analysis of the magnitude of these form factors is given in Appendix C. It is shown that

$$k_0 = 1 \quad (10a)$$

$$-\frac{d\phi}{df} = (1 + k_2 + k_3) \geq 0 = k_1 \text{ at } f = 0 \quad (10b)$$

$$-\frac{d\phi}{df} = 1 - k_2 - 2k_3 \geq 0 = k_5 \text{ at } f = 1 \quad (10c)$$

Other criteria for the validity of the form factors are given also.

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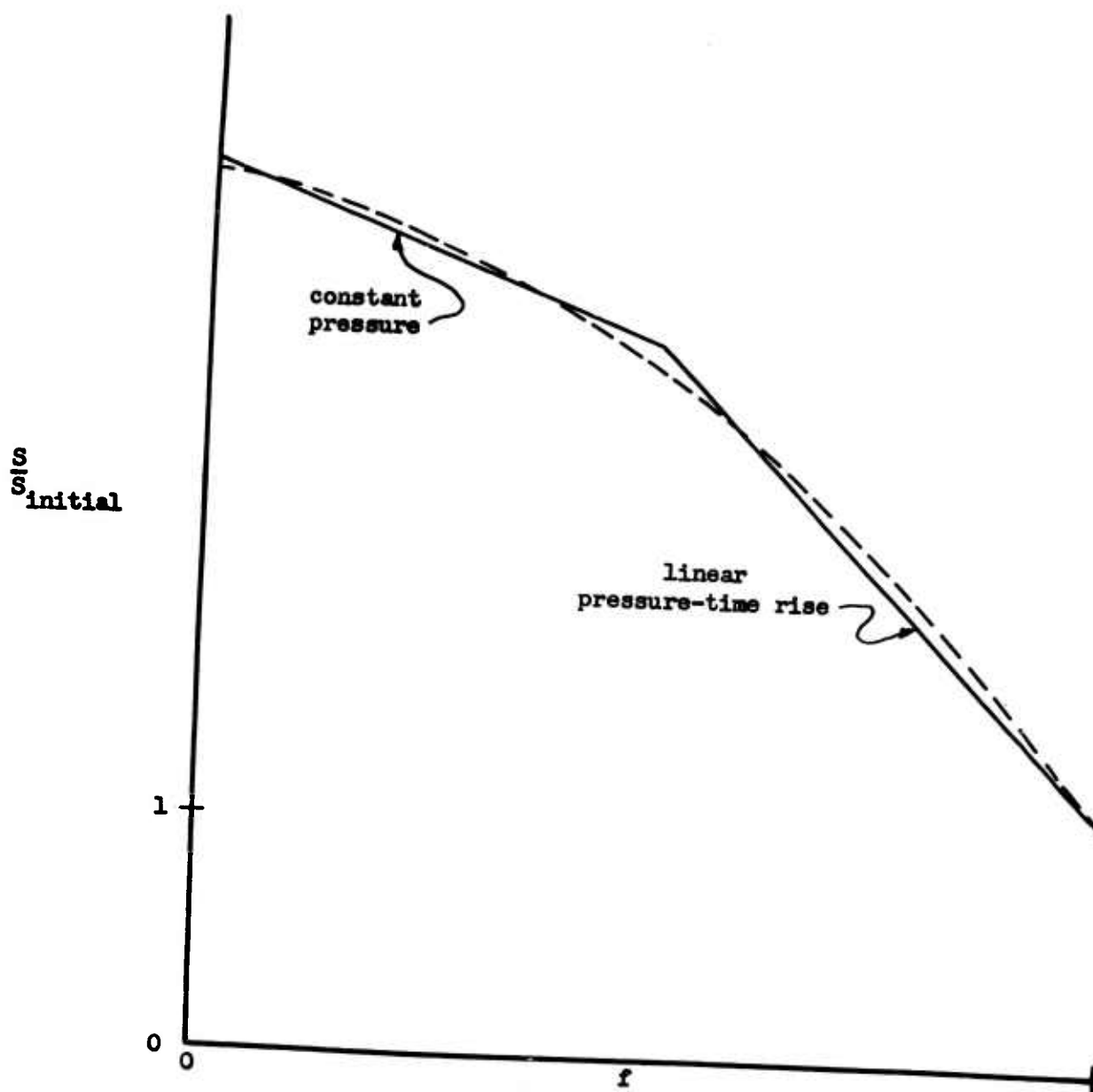


FIG. 2 VARIATION OF SURFACE WITH FRACTIONAL WEB

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A solution of Eq. (8) for the cubic equation is given in Appendix D.

D. Analysis of Experimental Data

Frankford Arsenal has obtained reproducible performance of a caliber .60 weapon using pyrotechnic ignition. Pressure-time data from these firings were furnished by the Arsenal and a ballistic analysis was made. Typical pressure-time curves from these data are shown in Fig. 3.

Representative experimental p-t data were double integrated to obtain velocity-time, travel-time and pressure-travel data.

Combining Eq. (1) and (2), the gas production function ϕ can be obtained in terms of p, v and λ .

$$\phi = \frac{(\bar{\gamma} - 1) \frac{m'v^2}{CF} + \frac{P}{F\Delta_o} (\lambda - \frac{\Delta_o}{\rho})}{1 + aP/F} \quad (11)$$

Thus experimental data can be compared with theoretical by two methods; pressure-travel data and $\phi(V)$ or $\phi(f)$ data.

1. Quadratic Form Function

A quadratic form function of the type

$$\phi = 1 - (1+k_2)f + k_2f^2 \quad (12)$$

was used and values of k_2 , σ and $\bar{\gamma}$ were determined which would match the experimental data. The results of several calculations are shown in Figs. 4, 5 and 6. In Fig. 6, it is seen that the maximum pressure and the area under the curve are closely approximated for values of $k_2 = 0.89$, $\sigma = 0.8$ and $\bar{\gamma} = 1.4$.

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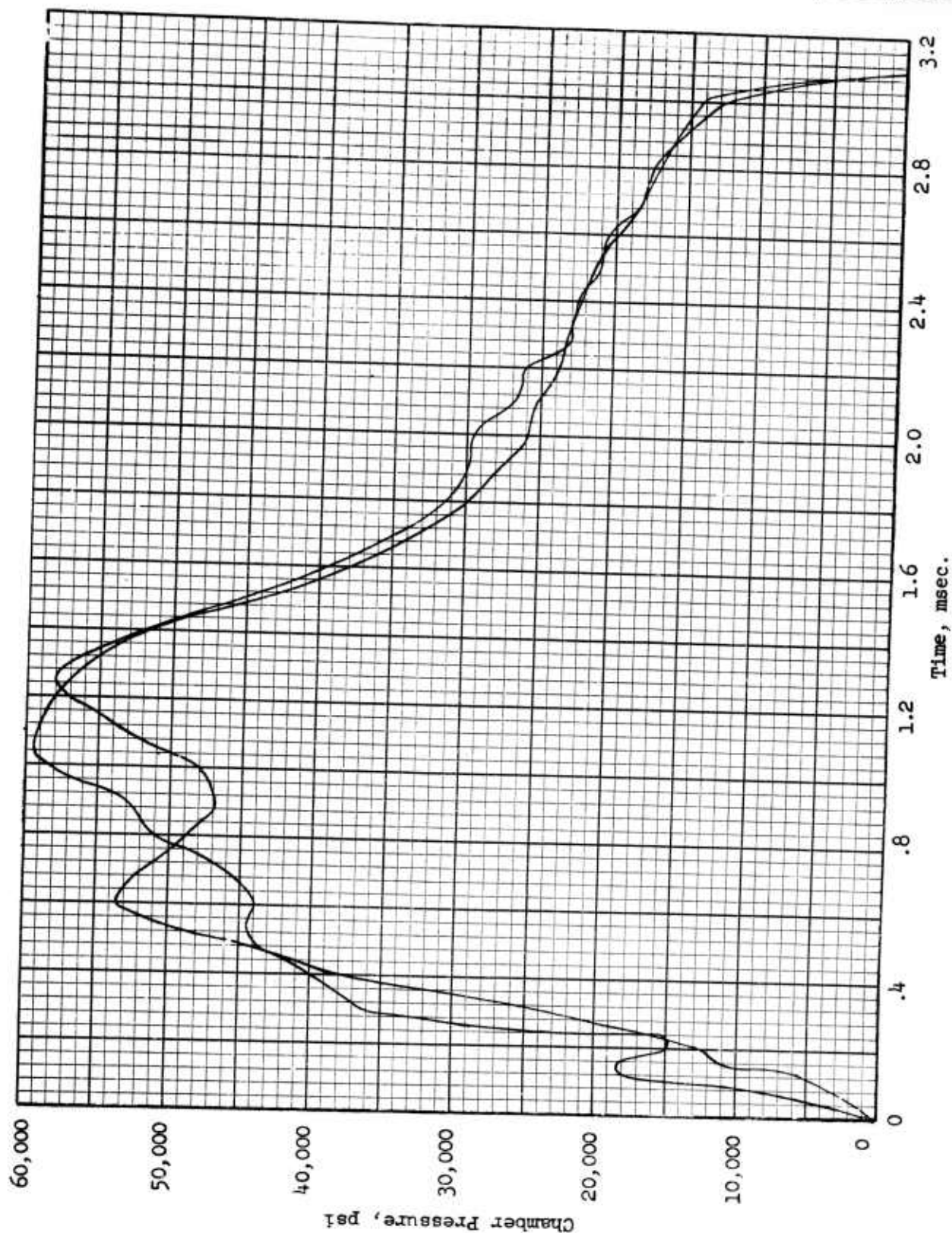


FIG. 3 REPRESENTATIVE PRESSURE-TIME DATA

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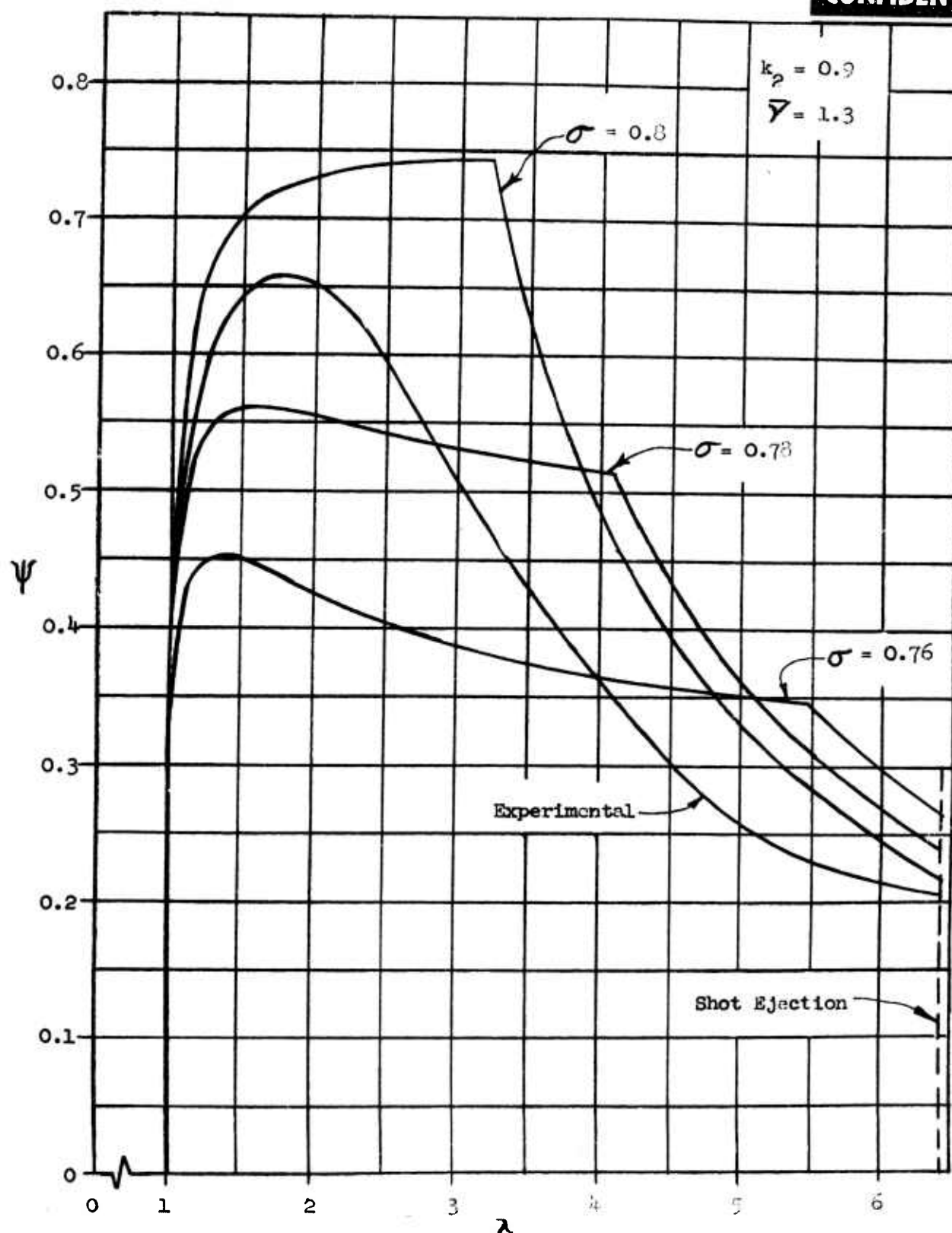


FIG. 4 EXPERIMENTAL AND CALCULATED PRESSURE-TRAVEL CURVES
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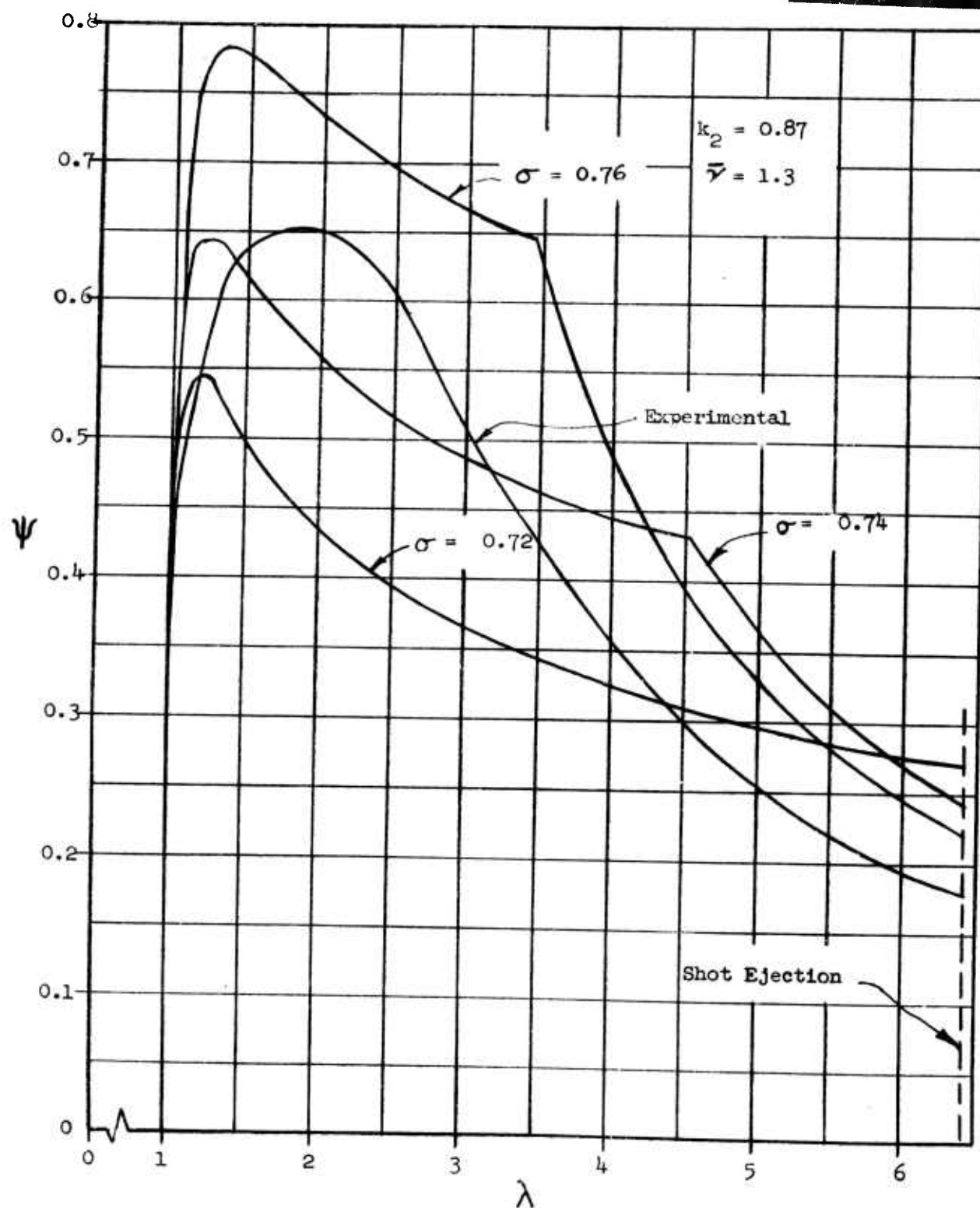


FIG. 5 EXPERIMENTAL AND CALCULATED PRESSURE-TRAVEL CURVES
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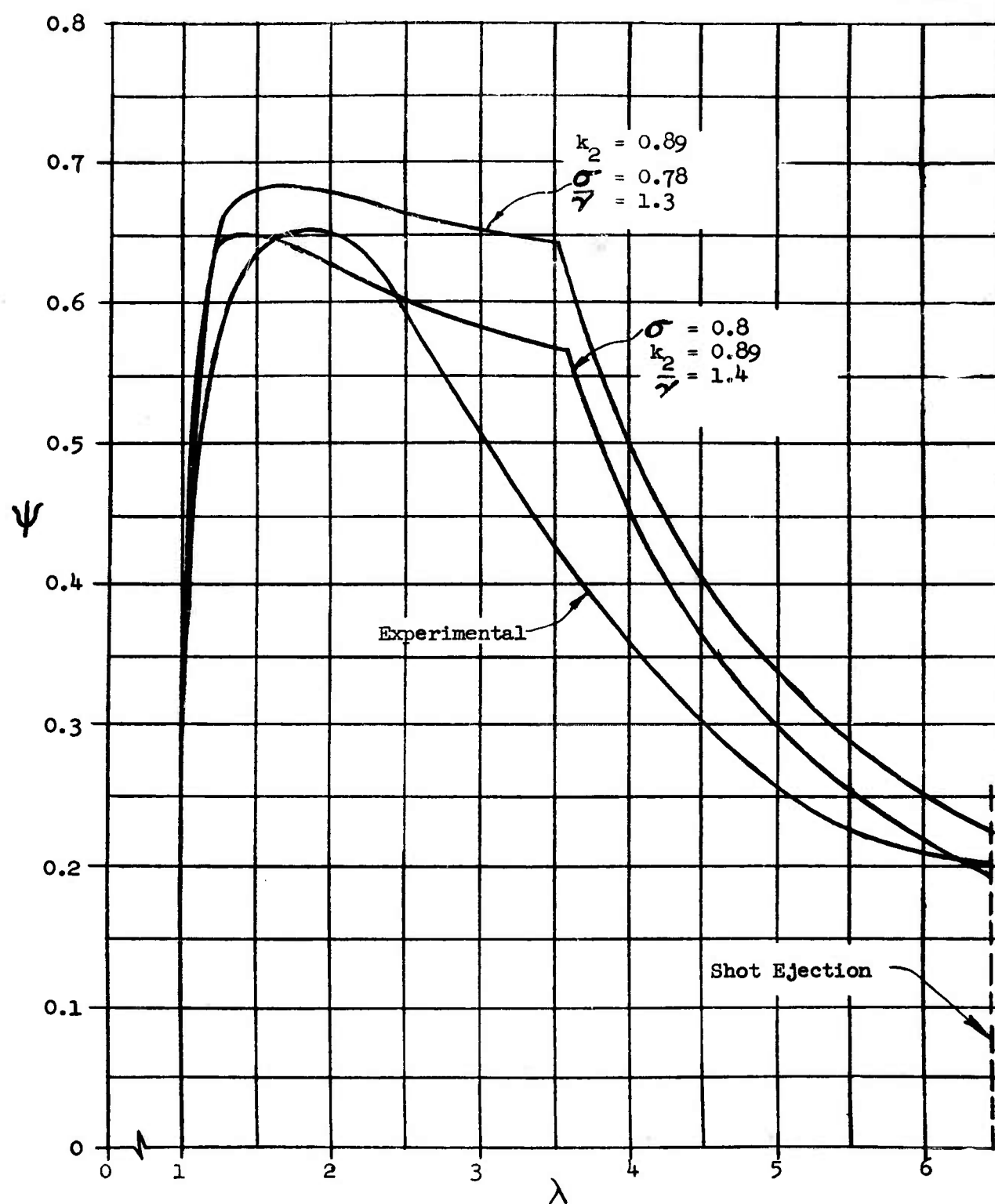


FIG. 6 EXPERIMENTAL AND CALCULATED PRESSURE-TRAVEL CURVES
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For the calculations and in analyzing the experimental data the impetus of the propellant was taken as 334,000 ft and η was assumed to be 30 in³/lb.

The constant k_2 in the quadratic form function is an indication of the progressivity of the propellant since

$$S \propto \frac{d\phi}{dt} \quad (13)$$

and

$$\frac{S_b}{S_1} = \frac{1+k_2}{1-k_2} \quad (14)$$

The modified specific heat ratio, $\bar{\gamma}$, reflects the energy in the gases since

$$E_o = \frac{NF}{\bar{\gamma} - 1}$$

Therefore a high value of $\bar{\gamma}$ means that there is less energy in the generated gas. These factors are apparent in Figs. 4, 5 and 6. An increase in k_2 tends to flatten out the pressure-travel area and give a lower peak for the same linear burning rate ϕ . The increase in $\bar{\gamma}$ causes the tail end of the curve to become lower and is particularly effective after all burnt.

A comparison of the ϕ - V relationship obtained from experimental data with values calculated from the quadratic form function is shown in Fig. 7. The individual points on the dashed are experimental values while the solid curves represent matching data from a quadratic form function. These data show that ϕ function follows a quadratic form

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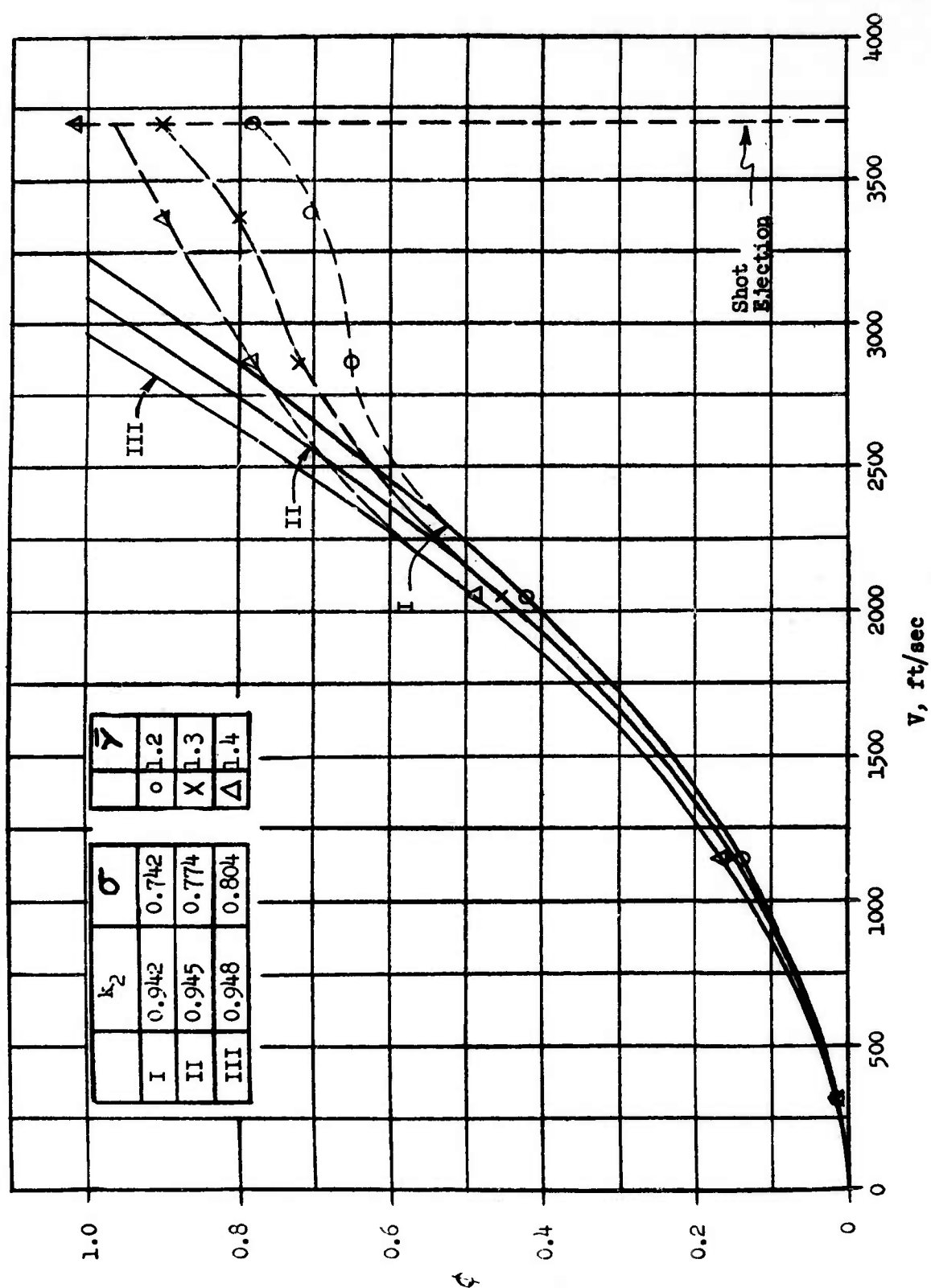


FIG. 7 QUADRATIC FIT TO EXPERIMENTAL GAS PRODUCTION

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to a velocity of 2000 fps where the experimental data shows a pronounced departure from the quadratic form with a decrease in slope. The matching values show that for $\bar{\gamma} = 1.4$, $k_2 = 0.948$ and $\sigma = 0.804$. This agrees with the pressure-travel curve except for the value of k_2 .

Analysis of the experimental data using $\bar{\gamma} = 1.4$ showed that all burnt occurs at shot ejection and $\sigma = 0.65$. However, it is shown that good agreement between experimental and theoretical curves can be obtained by compromising on the quadratic form factor and assuming a higher burning rate factor.

2. Cubic Form Function

Ballistic calculations were made using the cubic form function as expressed by Eq. (9). Values for k_2 and k_3 were determined from experimental data using the criteria set forth in Eq. (10) group. Since k_5 represents the slope of the surface curve at initial conditions, a value of 0.05 was assigned for the first calculations. The results of these calculations are shown in Fig. 8. These calculations show that the shape of the curve, maximum pressure and area under the curve can be closely approximated with

$$\begin{aligned} k_5 &= 0.05 \\ k_2 &= -0.528 \\ k_3 &= 0.739 \\ \sigma &= 0.76 \end{aligned}$$

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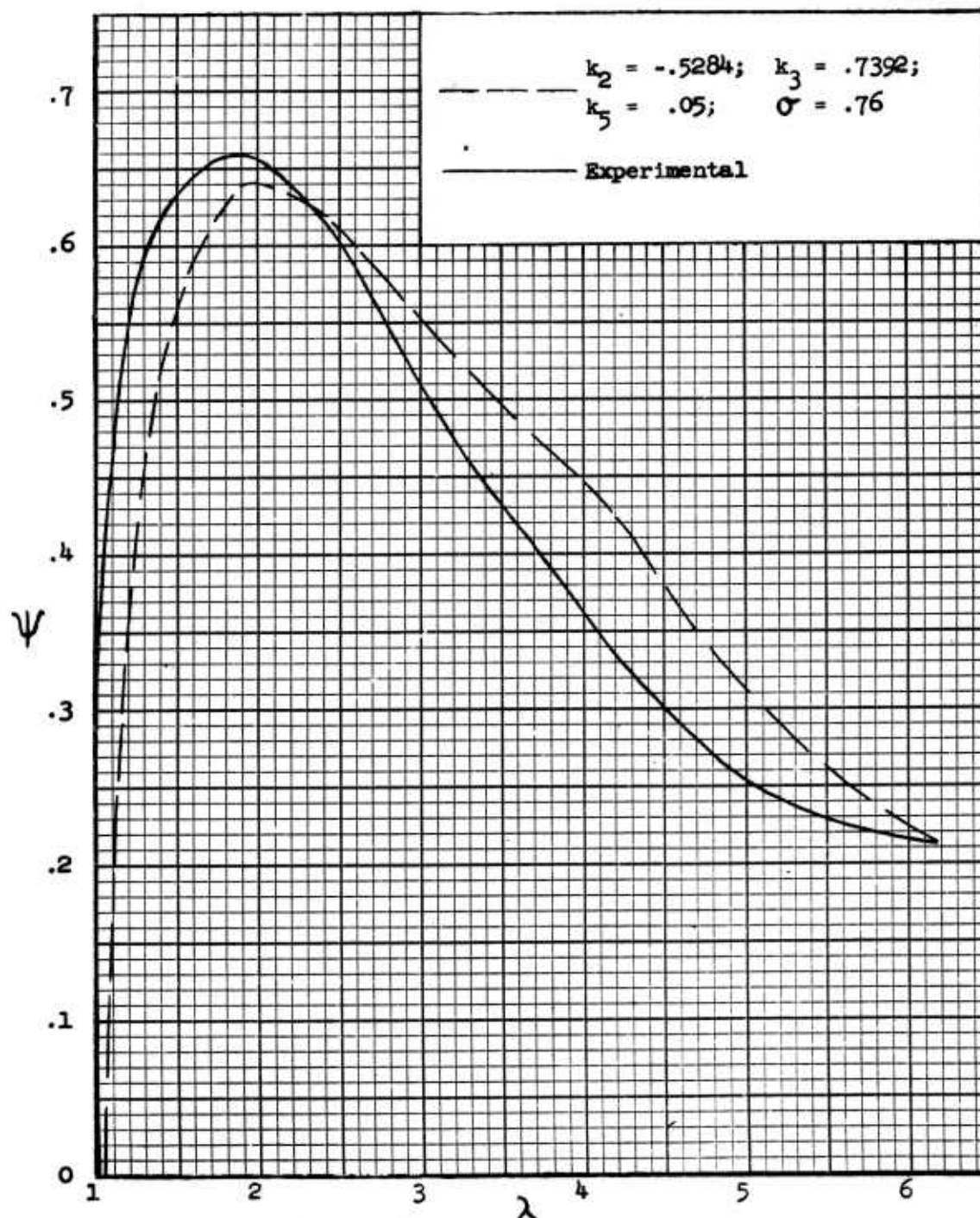


FIG. 8 COMPARISON OF CALUCLATED AND EXPERIMENTAL PRESSURE-TRAVEL DATA USING CUBIC FORM FUNCTION

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The error in maximum pressure is less than 4 per cent and the muzzle velocity from integration is only 1.2 per cent higher.

This curve is shifted in time from the experimental curve. A higher value for the initial slope, k_5 , was chosen in order to increase the rate of pressure rise. The initial slope for the quadratic form was determined to be 0.11 and a value of 0.1 was chosen for the next set of calculations. The results shown in Fig. 9 indicate that this results in a sharper peak but that the deviation in peak pressure is small and a compromise on values of k_2 , k_3 , and σ can keep the deviations to less than 5 per cent. This data also uses $\sigma = 0.69$ which is close to the experimentally determined value of 0.65.

Either the quadratic form function or the cubic equation may be used for calculating the pressure, velocity and travel for a desired weapon based on the performance of the caliber .60 weapon. Unfortunately, data available on other weapons were insufficient for good analysis and comparison purposes.

A comparison of the σ -f relation as determined from experimental data with data calculated by quadratic and cubic form functions is shown in Fig. 10. The cubic form function shows somewhat better correlation but is more difficult to use in the ballistic equations.

If electronic computers are available, the added complexity of the cubic form function is unimportant, however, it does involve the evaluation of one additional constant for the form function. Since

for engineering applications, adequate agreement of maximum pressure,

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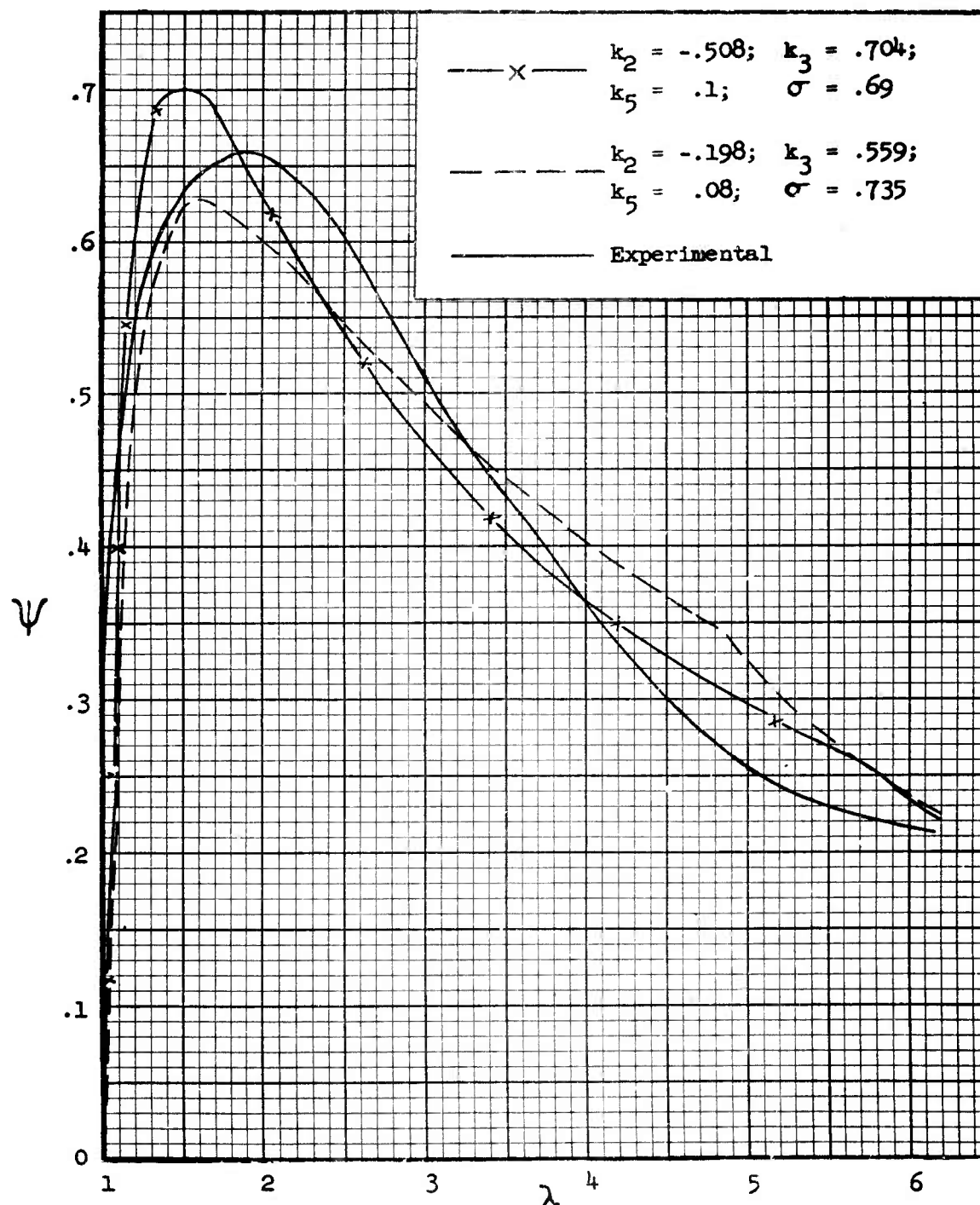


FIG. 9 COMPARISON OF CALCULATED AND EXPERIMENTAL PRESSURE-TRAVEL DATA USING CUBIC FORM FUNCTION

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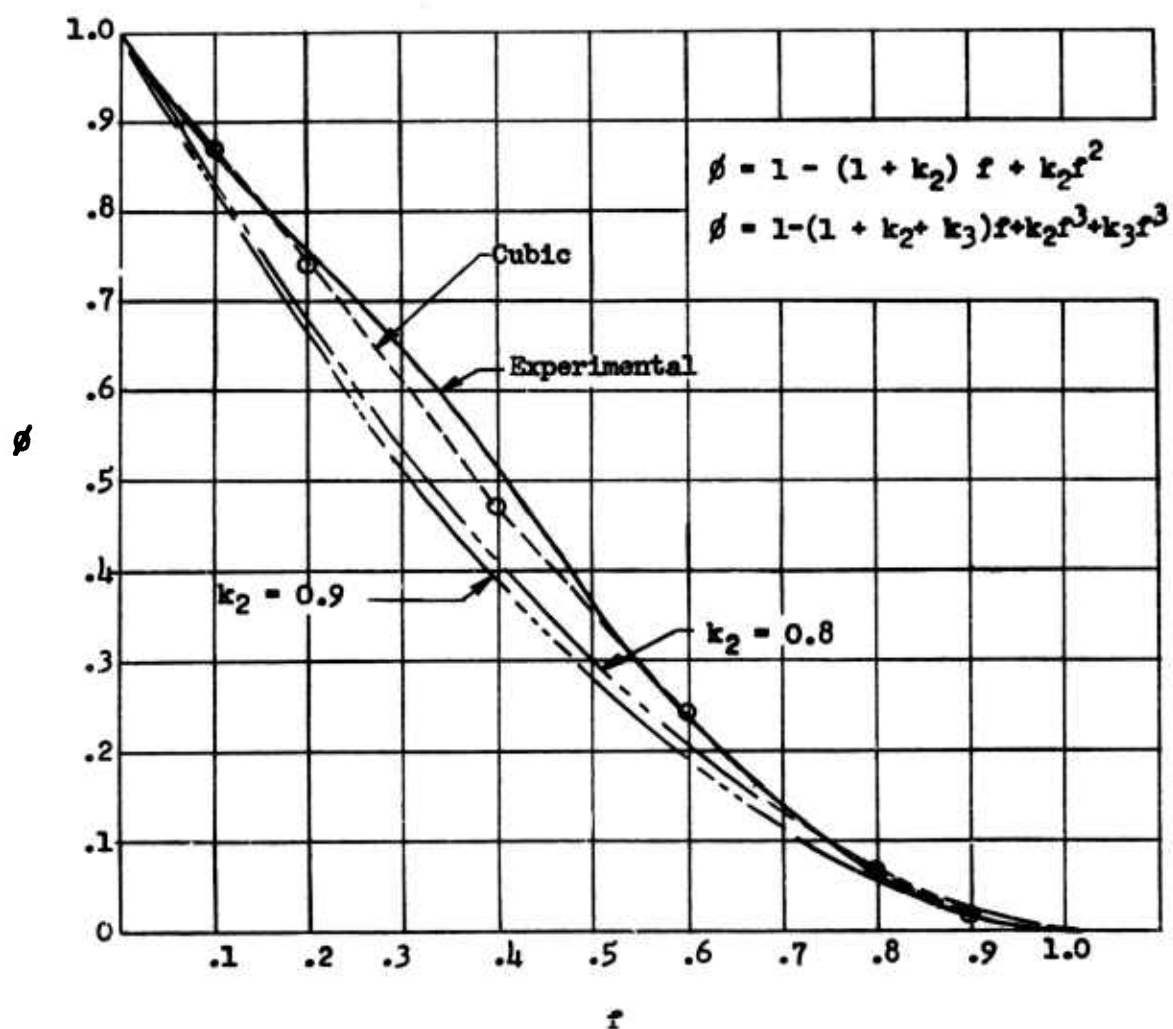


FIG. 10 COMPARISON OF ϕ VS. f
FROM EXPERIMENTAL AND CALCULATED DATA

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muzzle velocity and travel can be obtained with a quadratic function, use of the cubic probably cannot be justified on the basis of slightly better correlation.

3. Effect of Charge to Projectile Mass Ratio

Frankford Arsenal personnel fired a series of rounds in which the charge was held constant at the established value and the projectile mass was reduced, yielding a variation of the ratio of charge to projectile mass of as much as 2 to 1. Representative pressure-time data for each C/M are shown in Fig. 11. These data show that the magnitude of the ratio has negligible effect on the maximum pressure or the general shape of the curve. The smaller projectile mass results in a higher velocity and hence a shorter time to shot ejection and greater expansion rate. These data were analyzed also on the basis of the relationship between ϕ , the fraction of gas formed and f , the fictitious fraction of the web consumed. It is assumed that $f = 1 - (v/v_p)$. These data are shown in Fig. 12. It is apparent from these data the charge to mass ratio and the resulting changes in expansion rate (or projectile velocity) does not change the $\phi - f$ relationship.

4. Discussion of Data Evaluation

Two parameters in the ballistic equations are not well defined by experimental data to date. These are the co-volume, η and the ratio of specific heats, γ . Covolumes reported in the literature vary considerably from a maximum of 38 in³ per lb to a low of 21.5

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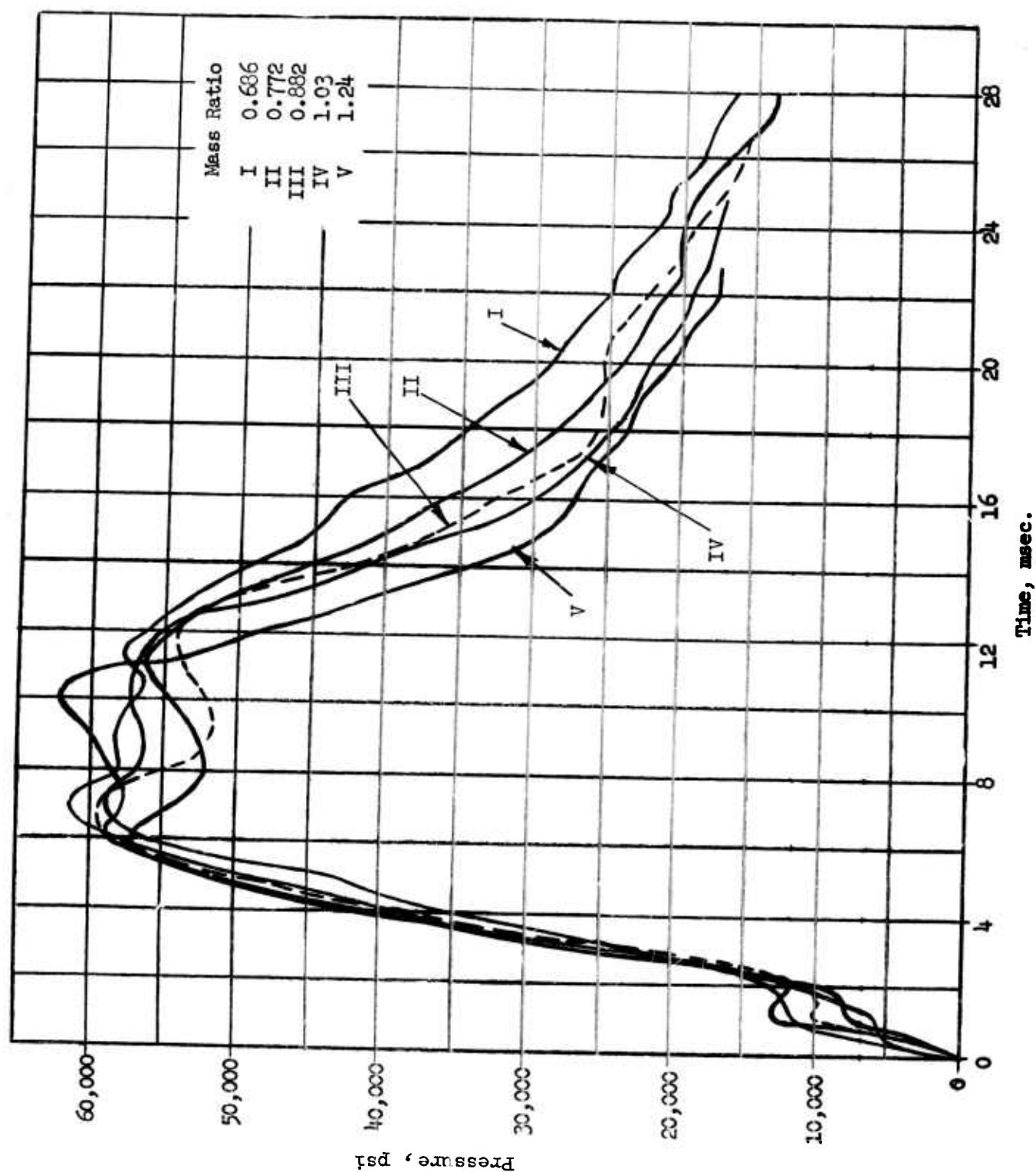


FIG. 11 REPRESENTATIVE PRESSURE-TIME DATA FOR VARIOUS CHARGE TO MASS RATIOS

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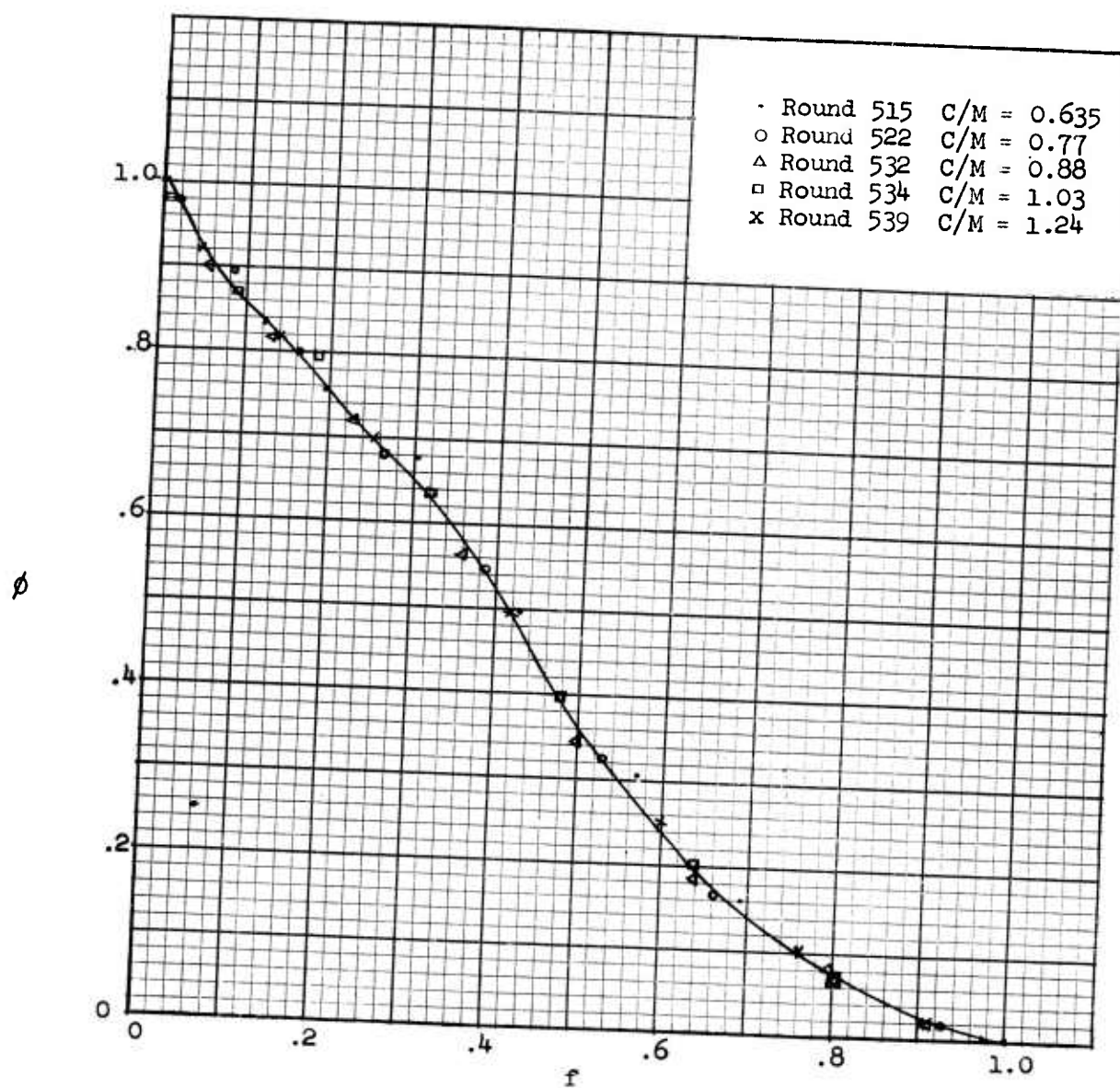


FIG. 12 EXPERIMENTALLY DETERMINED ϕ - f RELATIONSHIP

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in³ per lb. depending on the assumed composition of the products of combustion. Using 38 in³ per lb. resulted in extremely high calculated pressures and a value of 30 in³/lb was finally selected as giving reasonable results. This value has been reported from experiments also.

From literature it would be expected that γ should be between 1.2 and 1.3. An attempt to determine $\bar{\gamma}$ from experimental data failed because it was not possible to obtain sufficient information after all burnt. In the ballistic calculations $\bar{\gamma} = 1.4$ was used because it gave good correlation with the experimental p-x data.

The first set of data analyzed gave excellent results. Travel and velocity determined by integration agreed with indicated shot ejection time and muzzle velocity. The second set of data, for the rounds fired with varying charge to mass ratios, showed discrepancies. Rounds fired with the same charge and projectile mass as the previous data showed longer shot ejection times. When integrated, however, the data indicated shot ejection prior to the signal on the pressure trace and was in agreement with the earlier data. It was assumed, therefore, that the muzzle blanking unit was not recording properly and the time scale obtained from integration was used for all rounds.

The "malfunction" of the muzzle blank made it possible to obtain limited pressure data after shot ejection. Evaluation of the ϕ

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function from these data indicated that probably is close to the 1.2 value expected as indicated in Fig. 13. However, since a value of 1.4 must be used in the theoretical calculations to match experimental pressure-travel data, this has little effect on the choice of form function constants to be used if the assumption of delayed muzzle blanking is valid. It has been impossible to prove or disprove this assumption. Future experimental work should be carried out with sufficient tube length to insure the all burnt condition prior to shot ejection.

IV. ANALYSIS OF BURNING SURFACE FROM EXPERIMENTAL DATA

If Eq. 4 is solved for S, the variation of S with time which would give the measured pressure-time curve may be determined assuming this burning law.

Remembering that $N = C\phi$, Eq. 4 may be written

$$\frac{d\phi}{dt} = \frac{\rho_{SB}}{2C} p^n \quad (15)$$

from which

$$S = \frac{2C}{\rho_B} \frac{1}{p^n} \frac{d\phi}{dt} \quad (16)$$

Experimental values of $\phi(t)$ may be determined from Eq. (11).

The variation of ϕ thus calculated when $\bar{\gamma} = 1.2$ is shown in Fig. 13. The variation of S which would be required to give the measured pressure time curve for these conditions is shown in Fig. 14, for $n = 1$ and $n = 2.25$. The effect of assuming $n = 2.25$ is to flatten the first peak and accentuate the later maximum. This indicates that there is no large break-up of the liquid

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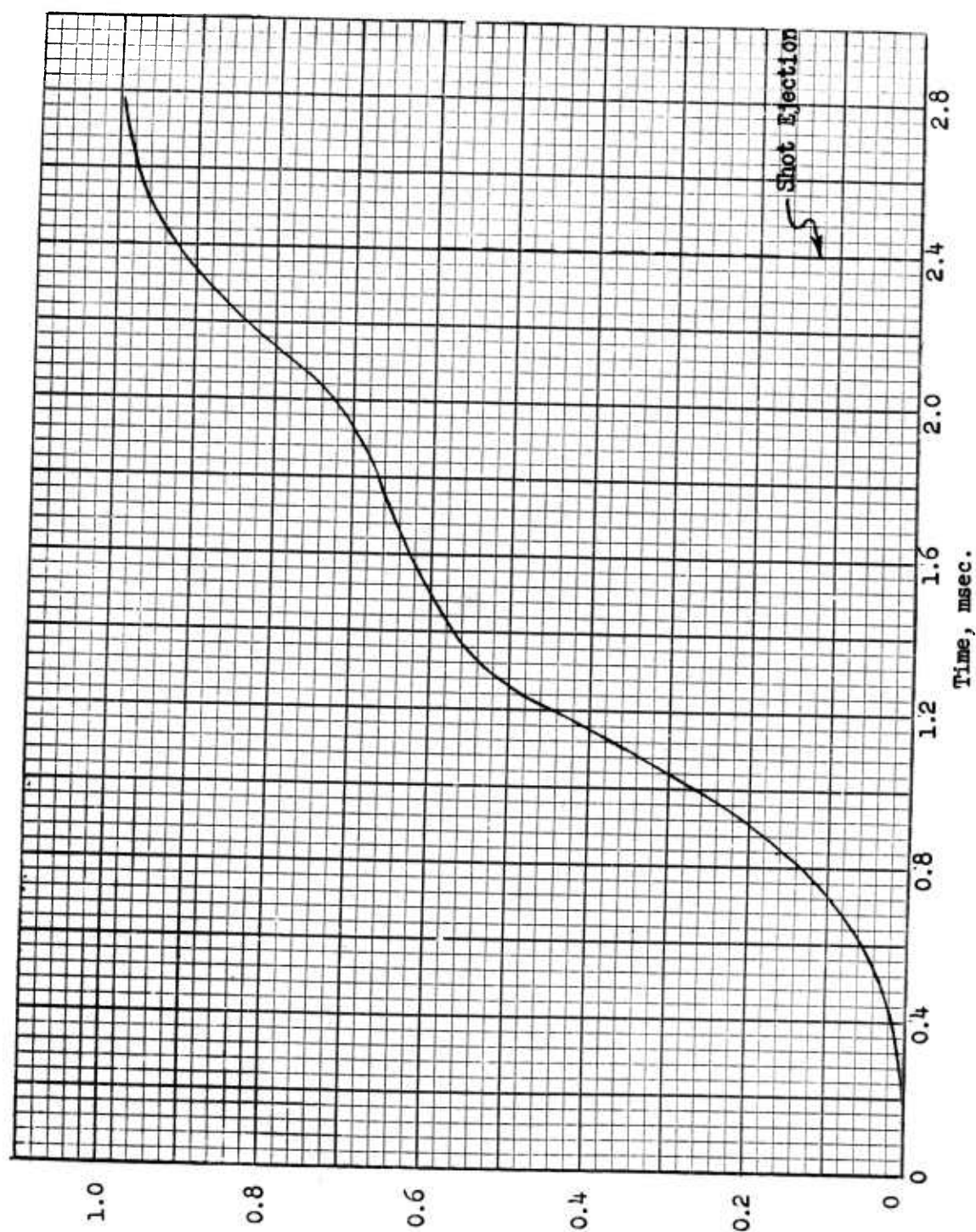


FIG. 13 VARIATION OF FRACTION BURNED WITH TIME

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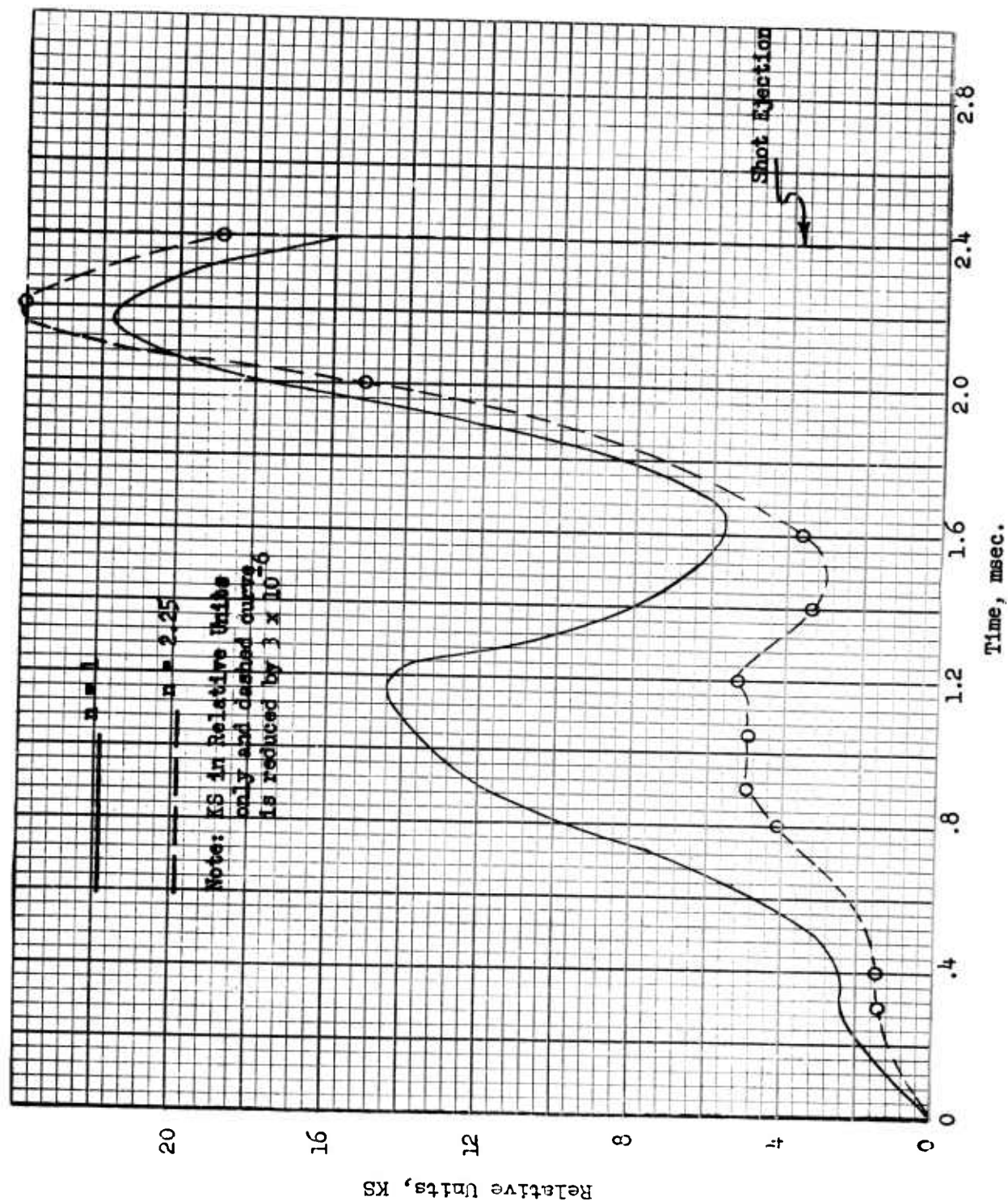


FIG. 14 VARIATION OF RELATIVE SURFACE AREA WITH TIME

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during the early burning period. The large increase in surface area occurs quite suddenly after the velocity has reached approximately 2800 fps. At this point, only about 35 per cent of the charge remains. The combination of increased chamber volume, smaller liquid charge, projectile velocity and physical properties of the liquid at this temperature and pressure have apparently reached a critical point allowing break-up.

Figure 15 shows a typical saddle-shaped pressure-time curve. The gas production function, ϕ , and the surface area variation are shown in Figs. 16 and 17. The surface area curve differs only slightly from Fig. 13 which was obtained from a relatively smooth pressure curve. Since both surface curves are obtained by measuring the slope of an experimentally determined curve, some discrepancy may be expected from the reading accuracy. The important information revealed by these data is the trend of the surface area at the time the saddle occurs in the pressure. In the time 0.6 to 1.0 msec, while the pressure is decreasing, the surface area is actually increasing appreciably. The first surface peak corresponds with the second pressure peak, however. Thus it would appear that the saddle shape results from a change in the linear burning rate or in the pressure exponent and not from a sharp decrease in burning surface. An analysis of the break-up forces would require knowledge of the viscosity, surface tension and density of the liquid at the prevailing temperature and pressure. The temperature of the liquid would be a function of the thermal properties which are unknown; estimates would be required.

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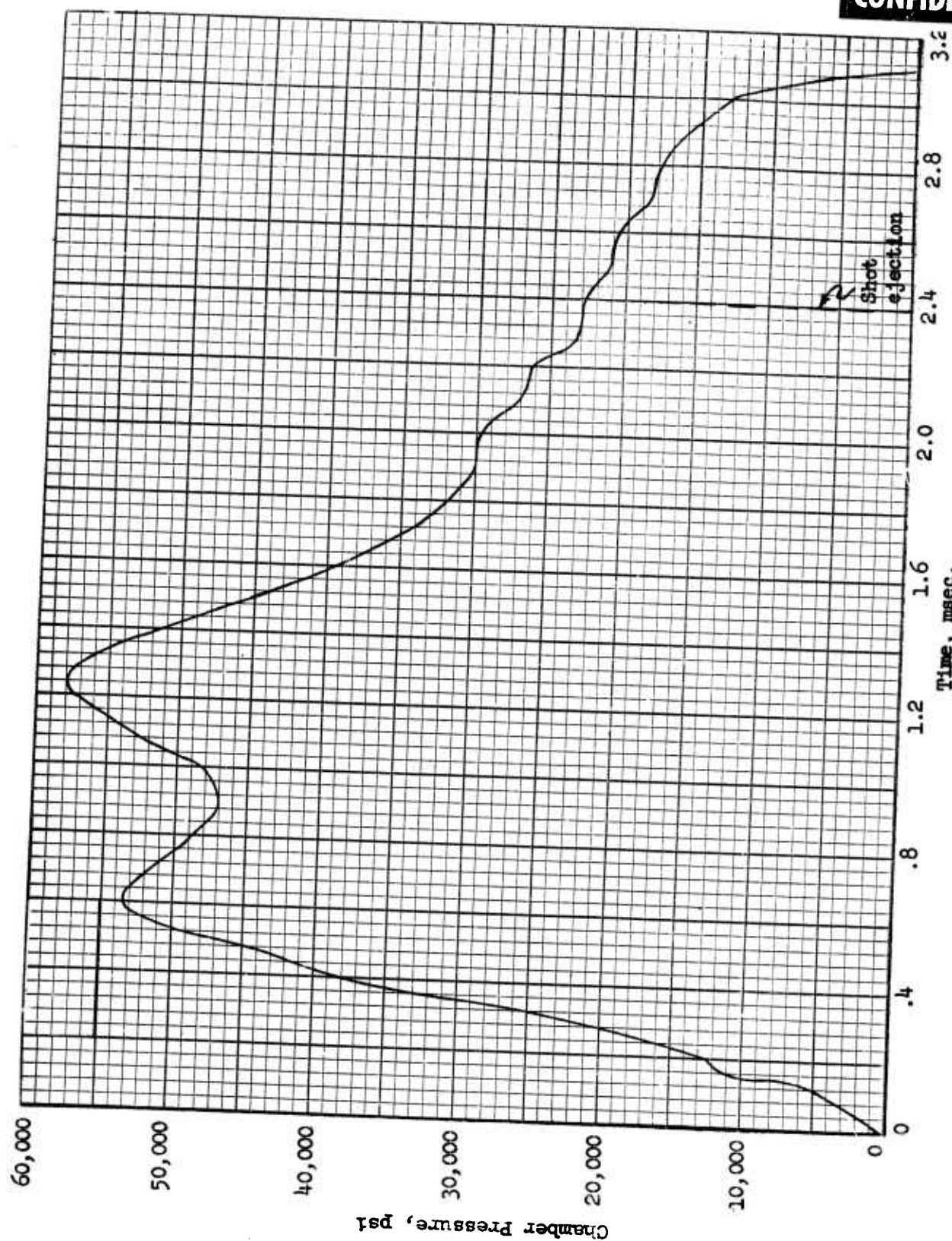


FIG. 15 TYPICAL SADDLE SHAPED PRESSURE-TIME DATA

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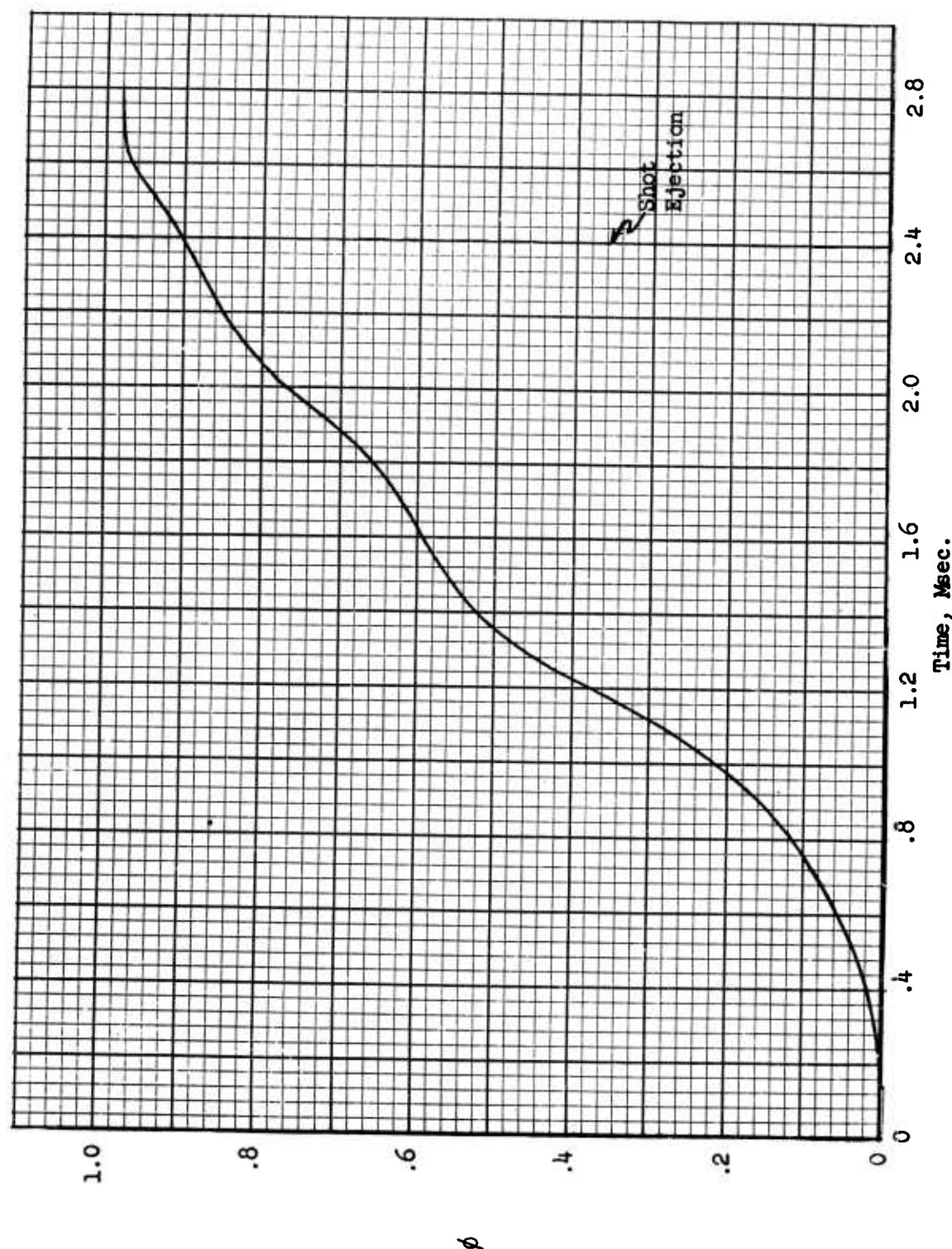


FIG. 16 VARIATION OF FRACTION BURNED WITH TIME

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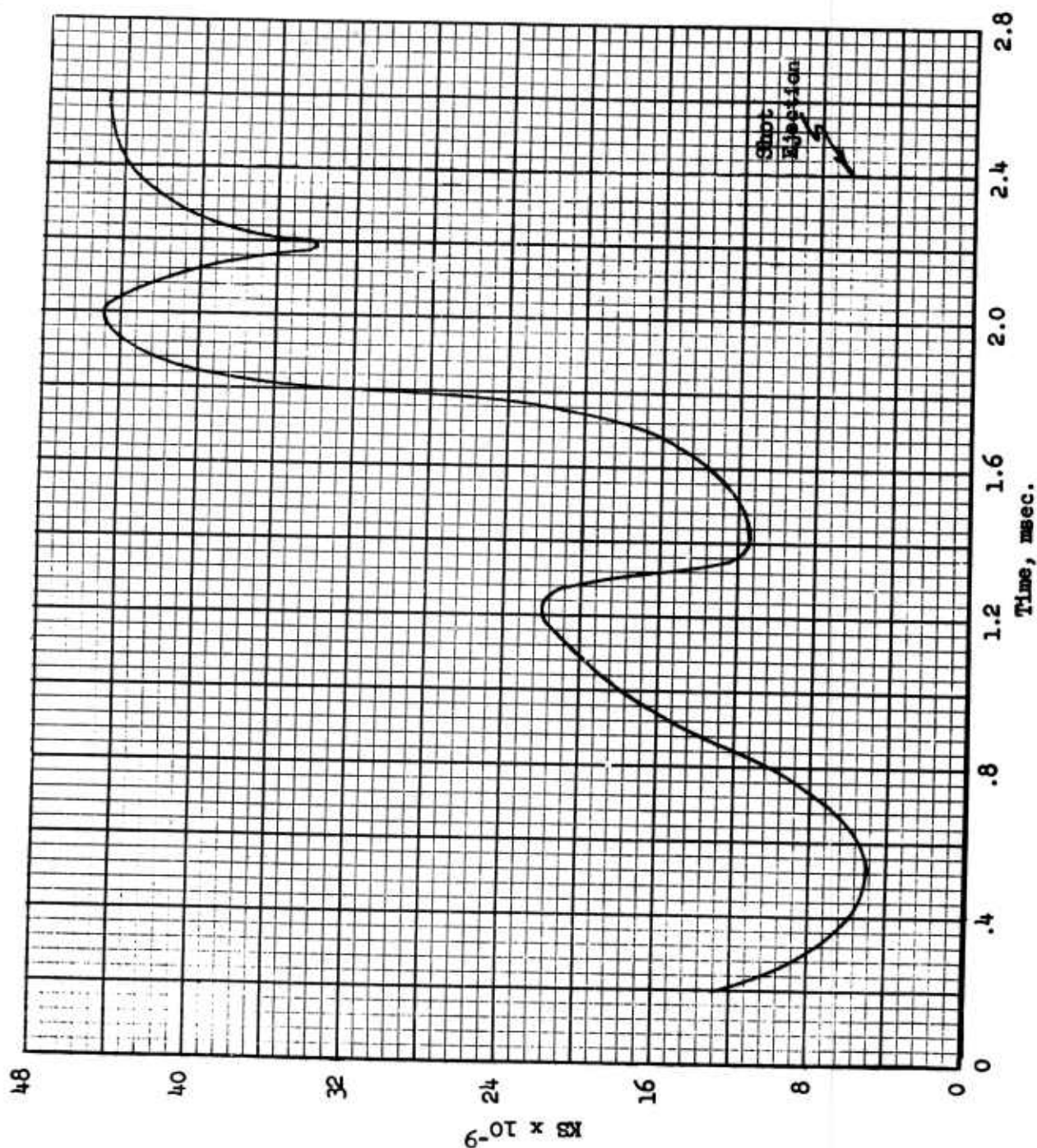


FIG. 17 VARIATION OF RELATIVE SURFACE AREA WITH TIME FOR SADDLE SHAPED
P-t CURVE

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V. CONCLUSIONS

The development of an interior ballistic theory for liquid monopropellants is by no means complete. A considerable number of closely controlled experimental firings are needed to provide sufficient data on the effect of various parameters. However, several conclusions may be drawn from the work reported here.

These are:

- A. The assumption of a linear burning rate equation with either a quadratic or a cubic form function may be used to predict the pressure-travel relationship. For the quadratic form function, $k_2 = 0.89$ and $\sigma = 0.80$. Close agreement is also shown for the cubic form function if $k_5 = 0.05$, $k_2 = -0.528$, $k_3 = 0.739$ and $\sigma = 0.76$. When $k_5 = 0.10$ good agreement can be obtained with $k_2 = -0.508$, $k_3 = 0.704$ and $\sigma = 0.69$. A modified specific heat ratio, $\bar{\gamma} = 1.4$ must be used in each case.
- B. The quadratic form function with the above constants and co-volume assumption provides adequate correlation for engineering purposes. Even with the benefit of electronic computers, the use of a cubic form function does not appear justified on the basis of these data.
- C. The form function as determined from the experimental data at high loading densities is unaffected by the ratio of charge weight to projectile mass, or therefore, the expansion ratio.
- D. Analysis of the burning surface by Eq(4) indicates that under the usual burning conditions large increases in the burning area do not occur until after peak pressure has been reached.

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- E. This analysis also indicates that the saddle shaped variation of the experimental pressure-time data may be due to a sudden decrease in either the constant or the pressure exponent in the burning equation and not due to a collapse of the available burning surface.
- F. More accurate determination of the co-volume is necessary to improve the ballistic calculations. The pressure is very sensitive to this parameter for weapons using high loading densities.

Further study of conclusions D and E should be made using experimental data in which other variations in the pressure time data are obtained. If the analysis is valid, it should be determined if the sharp increase in surface area after peak pressure can be correlated with some hydrodynamic and flow theories which have been advanced.

VI. RECOMMENDATIONS

Available experimental data is not sufficiently complete on any one system to result in an interior ballistic theory. Data obtained on one system can seldom be correlated in ballistic equations with data from another system because variations in several parameters generally occur. In order to fully evaluate the effects of chamber configuration, initial temperature viscosity, ignition, loading density, and charge to projectile mass ratio in the ballistic equations, experimental data must be obtained from a series of closely controlled experiments. The experimental weapon must be designed so that each of the parameters may be varied without changing any of the other parameters mentioned above.

The following specific recommendations are made:

- A. An experimental evaluation of the co-volume for the range of mixtures

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generally used. The evaluation should be made at pressures in excess of 50,000 psi in a closed bomb.

- B. Analytical and experimental evaluation of the heat losses during the ballistic cycle should be made.
- C. Conduct a closely controlled experimental program in which each of the factors suspected of influencing the performance may be investigated. Close tolerances on each factor must be held. Extreme care in selection of primers must be exercised. Temperatures should be closely controlled and reproducible.
- D. The firing program should be conducted using a gun barrel sufficiently long to insure that "all burnt" occurs well before shot ejection.
- E. Pressure data for several points in the chamber and barrel should be obtained.
- F. The experimental data should be thoroughly analyzed to determine what possible burning and ignition theories may apply and how they may be applied to the new data.

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APPENDIX A

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APPENDIX B

AN IDEALIZED PRESSURE-TIME CURVE

Consider a pressure-time curve which, from a given shot start pressure, rises linearly with time and then remains constant after attainment of a high pressure.

The assumed pressure variation is

$$\frac{P}{P_0} = 1 + bt \quad (B-1)$$

b = a constant

P_0 = shot start pressure

Equation of motion is written

$$AP = m' \frac{dv}{dt} \quad (B-2)$$

Integrate (B-2) using (B-1)

$$v = \frac{Ap_0}{m'b} \left[(bt) + \frac{1}{2} (bt)^2 \right] \quad (B-3)$$

Integrate (B-3) to yield travel

$$x - x_0 = \frac{Ap_0}{m'b^2} \left[\frac{(bt)^2}{2} + \frac{(bt)^3}{6} \right] \quad (B-4)$$

The energy and state equations are

$$NR (T_0 - T) = \frac{(\bar{\gamma} - 1)}{2} m' v^2 \quad (B-5)$$

$$P \left[AX - \frac{C-N}{\rho} - N\eta \right] = NRT \quad (B-6)$$

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Eliminate T between (B-5) and (B-6) using (B-1), (B-2), (B-3), and (B-4).
For the high volumetric loading densities used in liquid propellant, the approximation is made that $AX_0 = \frac{C}{\rho}$.

$$N = \frac{\frac{(\bar{\gamma}-1)}{2} \frac{A^2 P_o^2}{m'b^2} \left[bt + \frac{(bt)^2}{2} \right]^2 + \frac{A^2 P_o^2}{m'b^2} (1+bt) \left[\frac{(bt)^2}{2} + \frac{(bt)^3}{6} \right]}{F \left(1 + \frac{aP}{F} \right)} \quad (B-7)$$

The quantity $\frac{aP}{F}$ can be neglected. For representative propellant at 50,000 psi, $\frac{aP}{F} = 0.06$ and is, of course, smaller at lower pressure.

Equation of burning is rewritten

$$\frac{dN}{dt} = \rho S \left(\frac{B}{2} \right) P \quad (B-8)$$

Differentiate (B-7), neglecting $\frac{aP}{F}$, and equate to (B-8).

$$\frac{BS\rho}{2} = \frac{A^2 P_o^2}{m'Fb} \left[\bar{\gamma} \left(bt + \frac{b^2 t^2}{2} \right) + \frac{\left(\frac{b^2 t^2}{2} + \frac{b^3 t^3}{6} \right)}{(1+bt)} \right] \quad (B-9)$$

If the starting pressure is not unreasonably high and if the rate of pressure rise is high, as is the case, $bt \gg 1$ away from the origin.

Alter (B-9) accordingly.

$$\frac{BS\rho}{2} = \frac{A^2 P_o^2}{m'Fb} \left[\left(\bar{\gamma} + \frac{1}{2} \right) (bt) + \left(\frac{3\bar{\gamma}+1}{6} \right) (bt)^2 \right] \quad (B-10)$$

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From (B-3)

$$bt = -1 + \sqrt{1 + \frac{2m' bV}{AP_0}} \quad (B-11)$$

Away from the origin, (B-11) becomes

$$(bt) = \sqrt{\frac{2m' bV}{AP_0}} \quad (B-12)$$

Neglect the (bt) term in (B-10) and combine it with (B-12).

$$\frac{BS}{2} = (\bar{\gamma} + \frac{1}{3}) \left(\frac{AV}{F} \right) \quad (B-13)$$

Equation of burning is rewritten

$$S = - \frac{2C}{\rho W} \frac{d\phi}{dr} \quad (B-14)$$

Eliminate S between (B-13) and (B-14) and differentiate

$$\frac{d^2\phi}{dr^2} = \frac{\bar{\gamma} + \frac{1}{3}}{\sigma^2} \quad (B-15)$$

The constant pressure case is much simpler. The method is analogous to the rising pressure problem. Eq. (B-16) replaces (B-1).

$$P = P_1 \quad (B-16)$$

P_1 = constant pressure achieved in the firing.

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Integrate (B-2) using (B-16).

$$V - V_1 = \frac{AP_1}{m'} (t - t_1) \quad (B-17)$$

Subscript 1 is evaluated when the constant pressure epoch is first achieved.

Eliminate temperature from (B-5) and (B-6). Differentiate with respect to time, using (B-16), (B-17), (B-18), and (B-2) yielding (B-19).

$$V = \frac{dx}{dt} \quad (B-18)$$

$$\frac{dN}{dt} = \frac{\bar{\gamma} AP_1 V}{F \left(1 + \frac{1}{F}\right)} \quad (B-19)$$

Equate (B-19) to (B-8)

$$\frac{BS\phi}{2} = \frac{\bar{\gamma} AV}{F \left(1 + \frac{1}{F}\right)} \quad (B-20)$$

Combine (B-20) with (B-14) and differentiate

$$\frac{d^2\phi}{dt^2} = \frac{\bar{\gamma}}{\phi^2 \left(1 + \frac{1}{F}\right)} \quad (B-21)$$

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APPENDIX C

PERMISSIBLE FORM FACTORS
FOR THE CUBIC FORM FUNCTION

Consider the cubic form function

$$\phi = 1 - (1+k_2+k_3)f + k_2f^2 + k_3f^3 \quad (C-1)$$

From (B-14) and (C-1)

$$\frac{S}{C^2} = (1+k_2+k_3) - 2k_2f - 3k_3f^2 \quad (C-2)$$

C^2 = a positive constant.

The criterion for validity of the form factors is

$$S \geq 0 \quad \text{for} \quad 0 \leq f \leq 1 \quad (C-3)$$

Evaluate (C-2) at $f = 1$ and $f = 0$, using (C-3).

$$1 + k_2 + k_3 \geq 0 \quad (C-4)$$

$$1 - k_2 - 2k_3 \geq 0 \quad (C-5)$$

Add C-4 and C-5

$$k_3 \leq 2 \quad (C-6)$$

Multiply (C-4) by 2 and add to (C-5).

$$k_2 \geq -3 \quad (C-7)$$

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Equations (C-6) and (C-7) are necessary but not sufficient conditions for validity of the form factors.

The geometrical approach can be used. If (C-6) and (C-7) are valid, the surface is non-negative at the end points of the interval.

Equation (C-2) has either one maximum or one minimum and monotonically varies from the zero slope position. Therefore, knowing the surface at the end points and the surface and location of the zero slope position, (C-3) can be checked.

The zero slope position of the quadratic S curve and the surface there are found from (C-2). They are denoted by asterisks.

$$f^* = \frac{-k_2}{3k_3} \quad (C-8)$$

$$\frac{S}{C^2} = 1 + k_2 + k_3 + \frac{1}{3} \frac{k_2^2}{k_3} \quad (C-9)$$

$$\left. \frac{d^2 \left(\frac{S}{C^2} \right)}{df^2} \right|_* = -6k_3 \quad (C-10)$$

If $k_3 > 0$, a maximum behavior is indicated and therefore, the validity of (C-6), (C-7) and (C-11) show that the constants are valid.

$$k_3 > 0 \quad (C-11)$$

A M E R I C A N M A C H I N E & F O U N D R Y C O M P A N Y

C-2

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If $k_3 = 0$, the cubic degenerates into a quadratic which is of no interest here.

If $k_3 < 0$, a minimum behavior is exhibited. From (C-8), since $k_3 < 0$, $k_2 \leq 0$ places the minimum at $f \leq 0$, the proving validity of the constants.

If $\frac{-k_2}{3k_3} > 1$, the minimum lies to the right of the interval and the constants are proven. If $0 < f^* < 1$, (C-9) must be evaluated. If $S^* \geq 0$, the form factors are valid; if $S^* < 0$, the form factors are invalid.

Summing up, if values of k_2 and k_3 are examined for validity, proceed as follows;

1. Test (C-6) and (C-7)
 - a. If either is invalid, reject the form factors.
 - b. If both are valid, proceed to step 2.
2. Test (C-11).
 - a. If valid, the form factors are proven.
 - b. If $k_3 = 0$, the form function is quadratic and the solution is known.
 - c. If invalid, proceed to step 3.
3. If $k_2 \leq 0$, the form factors are proven
4. Evaluate f^* from (C-8)
 - a. If $f^* \geq 1$, the form factors are proven
 - b. If $f^* \leq 1$, proceed to step 5.
5. Compute S^*/C^2 from (C-9)
 - a. If $S^* \geq 0$, the form factors are proven
 - b. If $S^* < 0$, the form factors are rejected

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APPENDIX D

SOLUTION OF BALLISTIC EQUATION
USING A CUBIC FORM FUNCTION

The working ballistic equation is written

$$\frac{\bar{\gamma}-1}{2} I^2 = \phi - \frac{1}{2} \left(\lambda - \frac{\Delta_0}{\beta} - a \Delta_0 \phi \right) \frac{d}{d\lambda} (I^2) \quad (D-1)$$

and

$$f = f_0 - \sigma I \quad (D-2)$$

$$\phi = 1 - (1+k_2+k_3)f + k_2 f^2 + k_3 f^3 \quad (D-3)$$

$$\psi = \frac{d}{d\lambda} (I^2) \quad (D-4)$$

$$d\tau = \frac{dI}{\psi} = \frac{d\lambda}{2I} \quad (D-5)$$

k_2, k_3 = form factors

The above ballistic equations are subject to initial conditions.

$$\begin{array}{lll} I = 0 & f = f_0 & \phi = \phi_0 \\ \psi = \psi_0 & \lambda = 1 & \tau = 0 \end{array} \quad (D-6)$$

Combine (D-2) with (D-3)

$$\phi = \phi_0 + k_5 (\sigma I) + k_6 (\sigma I)^2 - k_3 (\sigma I)^3 \quad (D-7)$$

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D-1

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$$k_5 = (1+k_2+k_3) - (2k_2)f_o - (3k_3)f_o^2 \quad (D-8)$$

$$k_6 = k_2 + (3k_3)f_o \quad (D-9)$$

Combine (D-1) with (D-7)

$$I \frac{dI}{d\lambda} \left(\lambda - \frac{\Delta_o}{\rho} - a \Delta_o \phi \right) = F_1(I) \quad (D-10)$$

where

$$F_1(I) = \phi_o + k_5 (\phi I) + n_1 (\phi I)^2 - k_3 (\phi I)^3 \quad (D-11)$$

$$n_1 = k_6 - \frac{\gamma - 1}{2\phi^2} \quad (D-12)$$

Integrate by changing to the variable Z

$$Z = \lambda - \frac{\Delta_o}{\rho} - a \Delta_o \phi \quad (D-13)$$

Replace ϕ in (D-13) by (D-7). Differentiate and rearrange to yield
(D-14)

$$\frac{dI}{d\lambda} = \frac{1}{\frac{dZ}{dI} + F_2(I)} \quad (D-14)$$

where

$$F_2(I) = a \Delta_o \phi (k_5 + 2k_6 (\phi I) - 3k_3 (\phi I)^2) \quad (D-15)$$

Substitute (D-13) and (D-14) into (D-10)

$$\frac{dZ}{dI} + \left[\frac{-I}{F_1(I)} \right] Z = -F_2(I) \quad (D-16)$$

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Equation (D-16) is a well known form and is integrated from the initial condition. The variable Z is eliminated using (D-13).

$$\left[\lambda - \frac{\Delta_o}{\rho} - a \Delta_o \phi \right] J_c = \left[1 - \frac{\Delta_o}{\rho} - a \Delta_o \phi_o \right] + \int_0^I -F_2(I) J_c dI \quad (D-17)$$

$$J_c(I) = \exp \int_0^I \frac{-I}{F_1(I)} dI \quad (D-18)$$

Equation (D-17), with ϕ expressed in terms of I as given by (D-7), is the desired velocity-travel curve. The pressure is found by combining (D-1) and (D-4) to yield (D-19)

$$\psi = \frac{2 \left[\phi - \left(\frac{\bar{\gamma}-1}{2} \right) I^2 \right]}{\lambda - \frac{\Delta_o}{\rho} - a \Delta_o \phi} \quad (D-19)$$

The time may be found by graphically or numerically integrating (D-5).

As an aid to computation, (D-18) is integrated. The partial fraction expansion of the integrand of (D-18) may be found.

$$\frac{-I}{F_1(I)} = \frac{A_1}{I - I_1} + \frac{A_2}{I - I_2} + \frac{A_3}{I - I_3} \quad (D-20)$$

where I_1 , I_2 , and I_3 are the 3 roots of $F_1(I)$, which can be found by well known methods

$$A_1 = \frac{-I_1}{k_5 \omega + 2n_1 \omega^2 I_1 - 3k_3 \omega^3 I_1^2} \quad (D-21)$$

$$A_2 = \frac{-I_2}{k_5 \sigma + 2n_1 \sigma^2 I_2 - 3k_3 \sigma}$$

$$A_3 = \frac{-I_3}{k_5 \sigma + 2n_1 \sigma^2 I_3 - 3k_3 \sigma}$$

From (D-18) and (D-20)

$$J_c = \left[1 - \frac{I}{I_1} \right]^{A_1} \left[1 - \frac{I}{I_2} \right]$$

The integral in (D-17) can be evaluated numerically the procedure evaluating (D-24) is considered too tedious numerically integrated for J_c .